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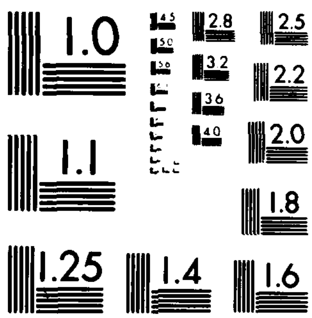

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ELECTROMAGNETIC COUPLING TO AN INFINITE WIRE THROUGH A SLOT IN A CONDUCTING PLANE

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The Problem of electromagnetic coupling through a slot-perforated conducting plane to an infinitely long wire is considered. It is treated as a boundary value problem and is formulated by use of the equivalence principle and image theory as a pair of coupled operator equations. The operator equations are solved simultaneously by means of the method of moments. The equivalent magnetic current on the slot and the electric current on the wire are obtained. The equivalent circuit seen by the transmission line mode on the

20. Abstract continued.

wire is derived for the aperture-to-wire coupling system. Computer programs for solving this problem are presented and described. Some numerical examples are given and plotted.

CONTENTS

PART ONE

GENERAL THEORY

I. INTRODUCTION

The problem of electromagnetic excitation of a wire through an aperture-perforated conducting plane has been studied in several papers. Kajfez [1] derived the equivalent sources and traveling current on the wire excited by a small circular aperture, but he neglected the interaction of the wire on the electric field in the aperture. The axial distribution of magnetic current on the slot is sometimes markedly affected by the presence of the wire. Butler and Umashankar [2] considered the boundary-value problem of thin wire of finite length behind a slotted conducting palne, obtaining a coupled set of integro-differential equations, which they solved numerically. A later paper [3] by Umashankar and Wait treated an infinite cable placed behind a slot-perforated screen problem by a Fourier transform method. However, there are errors in their equations and in their computed results. For example, the term cos λz is missing from their Eq. (19d), and no transmission line component of current is apparent in their Fig. 5.

In this report we obtain the functional equations for the problem using the equivalence principle [4], and reduce these equations to matrix form by means of the method of moments [5]. The matrices are recognized as generalized network parameters, such as voltages, currents, admittances, and impedances, as described in [5] and [6].

We next specialize the equations to the narrow slot and infinite thin wire problem. As electric current expansion functions, we take

outgoing transverse electromagnetic (TEM) currents traveling on both semi-infinite halves of the wire, and triangle functions to represent the higher order modes in a finite region near the aperture. This results in a finite sized matrix for the infinite wire, so that the solution is similar to that for a finite wire.

Finally, in addition to excitation by a plane wave, we consider excitation by a TEM traveling wave on the wire. The perturbation due to the effect of the slot can be obtained from the outgoing TEM currents in both directions. The equivalent circuit parameters for the TEM wave are then calculated from these outgoing traveling waves. Once the equivalent circuit parameters are obtained, we can calculate the currents traveling on the wire for the case of a wire terminated by arbitrary loads.

II. FORMULATION OF THE PROBLEM

Figure 1 shows the problem to be considered and the coordinates and notation to be used. The infinite conducting plane, which has zero thickness, covers the entire, $z=0$ plane except for the aperture. The aperture is rectangular in shape with length b and narrow width w in the y and x directions, respectively. The plane divides the space into two regions, called region a and region b. There is an infinitely long wire whose axis is along the x-direction and at a distance $z=d$ and $y=y_c$ in region b. The excitation of the slot is a uniform plane wave incident from the region a. The problem is primarily that of finding the tangential electric field on the slot and the current on the wire. Secondarily, it is that of finding the parameters of the equivalent network seen by the TEM traveling wave on the wire.

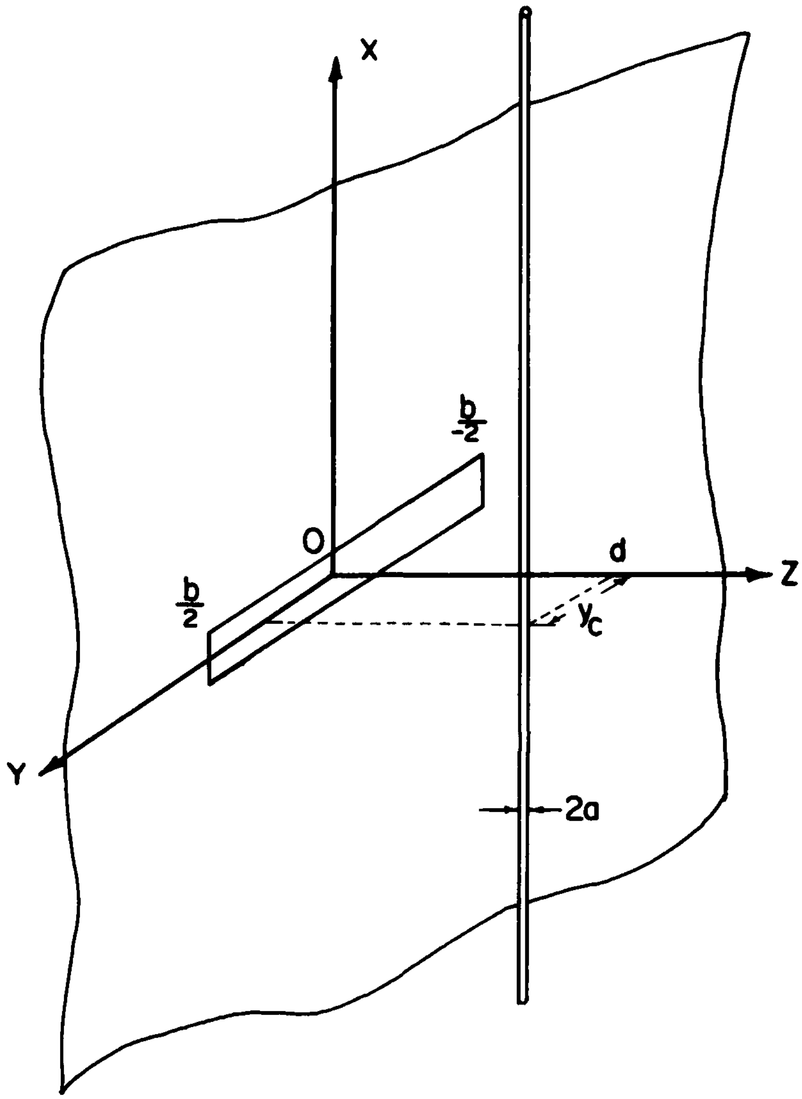


Fig. 1. Thin infinitely long wire behind a narrow slot perforated conducting plane.

We use the equivalence principle [4] to divide the original problem into two parts, as shown in Fig. 2. The field in region a remains unchanged if the slot is closed by a conductor and the equivalent surface magnetic current

$$\underline{M} = \underline{u}_z \times \underline{E} \tag{1}$$

is placed over the slot, where $\underline{E}$ is the electric field in the slot of the original problem and $\underline{u}_z$ is the unit vector in z direction. The incident wave $\underline{E}^i$, $\underline{H}^i$ must be kept in region a. This equivalence is shown in Figure 2a. The field in region b remains unchanged if the slot is closed by a conductor and the equivalent magnetic current $-\underline{M}$ is placed over the slot. The electric current I on the wire must be kept the same as the original problem. This equivalence is shown in Fig. 2b.

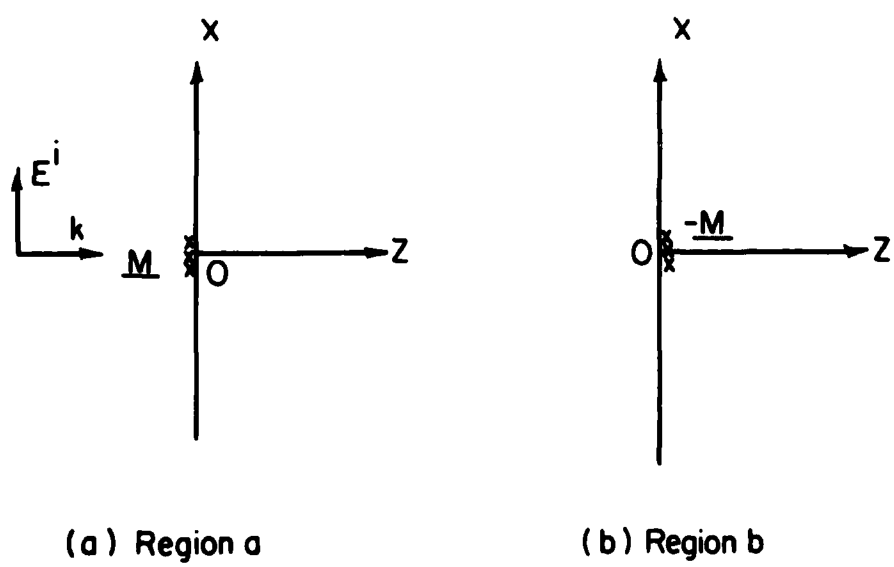


Fig. 2. The equivalent problem

The use of $\underline{M}$ in region a and $-\underline{M}$ in region b ensures the continuity of the tangential electric field across the slot. In addition, we have two more boundary conditions to enforce: (a) the tangential magnetic field must be equal on each side of the slot, and (b) the tangential electric field must vanish on the surface of the wire. These two conditions give us two equations from which we can calculate the unknowns $\underline{M}$ and $\underline{I}$.

The electric and magnetic fields in region a are given by

$$\underline{E}^a = \underline{E}^a(\underline{M}) + \underline{E}^i \tag{2}$$

$$\underline{H}^a = \underline{H}^a(\underline{M}) + \underline{H}^i \tag{3}$$

where $\underline{E}^a(\underline{M})$, $\underline{H}^a(\underline{M})$ are the fields from $\underline{M}$ in Fig. 2a, and $\underline{E}^i$, $\underline{H}^i$ are incident fields in region a. Note that all the fields are computed with the slot shorted. Similarly, we denote the electric and magnetic fields in region b to be

$$\underline{E}^b = \underline{E}^b(-\underline{M}) + \underline{E}^b(\underline{I}) \tag{4}$$

$$\underline{H}^b = \underline{H}^b(-\underline{M}) + \underline{H}^b(\underline{I}) \tag{5}$$

where $\underline{E}^b(-\underline{M})$, $\underline{H}^b(-\underline{M})$ are the fields from $-\underline{M}$ in Fig. 2b, and $\underline{E}^b(\underline{I})$, $\underline{H}^b(\underline{I})$ are the fields from $\underline{I}$ on the wire in Fig. 2b. Again, all the fields are computed with the slot shorted.

To enforce the two boundary conditions (a) and (b), we set

$$\underline{H}_t^a(\underline{M}) + \underline{H}_t^b(\underline{M}) - \underline{H}_t^b(\underline{I}) = -\underline{H}_t^i \qquad \text{over A} \tag{6}$$

$$\underline{E}_t^b(\underline{M}) - \underline{E}_t^b(\underline{I}) = 0 \qquad \text{over B} \tag{7}$$

where A and B denote the slot and the wire surfaces, respectively. The subscripts t denotes the tangential component over A and B. We have used

the linearity of the operators in deriving (6) and (7), by setting $\underline{H}_t^b(-\underline{M}) = -\underline{H}_t^b(\underline{M})$ and $\underline{E}_t^b(-\underline{M}) = -\underline{E}_t^b(\underline{M})$. Equations (6) and (7) are vector equations for determining the unknowns $\underline{M}$ and $\underline{I}$.

We now reduce (6) and (7) to matrix equations using the method of moments [5]. We define a set of expansion functions $\{\underline{M}_n,\ n=1,2,\ldots,NA\}$ and express the magnetic current as

$$\underline{M} = \sum_{n=1}^{NA} V_n \underline{M}_n \tag{8}$$

where V_n are coefficients to be determined. We define a set of expansion functions $\{\underline{I}_n,\ n=1,2,\ldots,NB\}$, and express the electric current as

$$\underline{I} = \sum_{n=1}^{NB} \alpha_n \underline{I}_n \tag{9}$$

where α_n are coefficients to be determined. We substitute (8) and (9) into (6), and (7), using the linearity of the operator and obtain

$$-\sum_{n=1}^{NA} V_n \underline{H}_t^a(\underline{M}_n) - \sum_{n=1}^{NB} V_n \underline{H}_t^b(\underline{M}_n) + \sum_{n=1}^{NB} \alpha_n \underline{H}_t^b(\underline{I}_n) = \underline{H}_t^i \tag{10}$$

over A, and

$$\sum_{n=1}^{NA} V_n \underline{E}_t^b(\underline{M}_n) - \sum_{n=1}^{NB} \alpha_n \underline{E}_t^b(\underline{I}_n) = 0 \tag{11}$$

over B. We define testing functions $\{\underline{\hat{M}}_m,\ m=1,2,\ldots,NA\}$ on A and $\{\underline{\hat{I}}_m,\ m=1,2,\ldots,NB\}$ on B, and the symmetric product

$$\langle \underline{F}, \underline{G} \rangle_s = \iint_S \underline{F} \cdot \underline{G}\, ds \tag{12}$$

where S represents the surface of A or B. We take the symmetric product of (10) with each $\underline{\hat{M}}_m$ on A to obtain

$$[Y^a + Y^b]\vec{V} + [T]\vec{I} = \vec{I}^i \tag{13}$$

where

$$[Y^a] = [\langle -\underline{\hat{M}}_m, \underline{H}_t^a(\underline{M}_n)\rangle_A] \tag{14}$$

$$[Y^b] = [\langle -\underline{\hat{M}}_m, \underline{H}_t^b(\underline{M}_n)\rangle_A] \tag{15}$$

The $[Y^a]$ and $[Y^b]$ are called generalized admittance matrices for region a and b, respectively. The matrix

$$[T] = [\langle \hat{\underline{M}}_m, \underline{H}_t^b(\underline{I}_n)\rangle_A] \tag{16}$$

is called the coupling matrix from B to A. The column matrix

$$\vec{I}^i = [\langle \hat{\underline{M}}_m, \underline{H}_t^i \rangle_A] \tag{17}$$

is called excitation vector. Finally, the column matrices

$$\vec{V} = [V_n] \tag{18}$$

$$\vec{I} = [\alpha_n] \tag{19}$$

are called generalized voltage and current vectors, respectively.

Next, we take the symmetric product of (11) with each $\hat{\underline{I}}_m$ to obtain

$$[\hat{T}]\vec{V} + [Z]\vec{I} = 0 \tag{20}$$

where

$$[\hat{T}] = [\langle \hat{\underline{I}}_m, \underline{E}_t^b(\underline{M}_n)\rangle_B] \tag{21}$$

is called coupling matrix from A to B, and

$$[Z] = [\langle -\hat{\underline{I}}_m, \underline{E}_t^b(\underline{I}_n)\rangle_B] \tag{22}$$

is called generalized impedance matrix for the wire.

We now have the matrix equations (13) and (20), from which we can solve for $\vec{V}$ and $\vec{I}$. Note that $[Y^a]$ and $[Y^b]$ are evaluated

for the slot as if the wire were not present, and are hence the same matrices as described in [6]. [Z] is evaluated for wire as if the slot were not present, that is, with the slot closed by a conductor. It can be evaluated in terms of currents plus their images radiating into free space.

III. EVALUATION OF THE MATRICES

Now we specialize the problem to a narrow rectangular slot and a thin wire, and evaluate all the matrices in equations (13) and (20). We use Galerkin's method, which means that

$$\hat{\underline{M}}_m = \underline{M}_m \qquad (m = 1,2,\ldots,NA) \tag{23}$$

$$\hat{\underline{I}}_m = \underline{I}_m \qquad (m = 1,2,\ldots,NB) \tag{24}$$

This will save us some computations. In particular, from reciprocity [4], we have

$$\langle \hat{\underline{M}}_m, \underline{H}_t(\underline{M}_n)\rangle_A = \langle \underline{M}_n, \underline{H}_t(\hat{\underline{M}}_m)\rangle_A$$

$$\langle \hat{\underline{I}}_m, \underline{E}_t(\underline{I}_n)\rangle_B = \langle \underline{I}_n, \underline{E}_t(\hat{\underline{I}}_m)\rangle_B$$

Substituting (23) and (24) into (14), (15) and (22), we find that

$$Y^a_{mn} = Y^a_{nm} \qquad \text{or} \qquad [Y^a] = [\tilde{Y}^a] \tag{25}$$

$$Y^b_{mn} = Y^b_{nm} \qquad \text{or} \qquad [Y^b] = [\tilde{Y}^b] \tag{26}$$

$$Z_{mn} = Z_{nm} \qquad \text{or} \qquad [Z] = [\tilde{Z}] \tag{27}$$

where "~" denotes the transpose of a matrix. Equations (25), (26) and (27)

indicate that the generalized admittance and impedance matrices are symmetrical matrices. Again, from reciprocity, we have

$$\langle \underline{\hat{I}}_m, \underline{E}_t^b(\underline{M}_n)\rangle_B = - \langle \underline{M}_n, H_t^b(\underline{\hat{I}}_m)\rangle_A$$

Substituting (23) and (24) into (16) and (21), we find that

$$\hat{T}_{mn} = - T_{nm} \qquad \text{or} \qquad [\hat{T}] = - [\tilde{T}] \tag{28}$$

Hence, we only need to evaluate one of the matrices [T] and $[\hat{T}]$.

(A) Evaluation of [Z]

First, we have to choose appropriate expansion functions for the current so that our results converge rapidly to save computation. We note that the wire and the complete conducting plane form a transmission line. We therefore expect that the presence of the slot on the plane will excite outgoing traveling TEM waves along the wire plus evanescent waves near the slot. We can expect that the TEM traveling waves will propagate to infinity without attenuation if the materials are loss-free. The evanescent waves, which are higher order modes on the transmission line, attenuate rapidly. Therefore, we assume that the evanescent waves will exist only over a short region near the slot. Beyond this region, the evanescent waves are so small that they do not contribute significantly to the field, and hence may be taken to be zero. It is therefore convenient to choose the following expansion functions for electric current I on the wire:

$$\underline{I}_1 = e^{-jk|x|}\underline{u}_x \qquad -\infty < x < \infty \tag{29}$$

$$\underline{I}_n = \underline{T}_n(x) \qquad n = 2, 3, \ldots \text{NB} \tag{30}$$

where k is the wavenumber of incident wave, $\underline{u}_x$ is the unit vector of x direction, and $\underline{T}_n(x)$ are triangle functions as defined in Appendix B. We can see that $\underline{I}_1$ represents the outgoing TEM mode, and $\underline{I}_n (n > 1)$ represent the evanescent currents near the slot. We here assume that the higher order modes exist only in the region $x < |L/2|$.

Now we can evaluate the elements of the generalized Z matrix. As a matter of fact, for $m \neq 1$, $n \neq 1$, it is the Z matrix of a wire whose length is L above a conducting plane, as is described in Appendix B. The results are

$$Z_{mn} = j\omega\mu Z_1 + \frac{1}{j\omega\varepsilon} Z_2 \tag{31}$$

where Z_1 and Z_2 are expressed in (B-18) and (B-19).

For m = n = 1, from (22) we have

$$Z_{11} = - \langle \underline{I}_1, \underline{E}_t(\underline{I}_1) \rangle_B \tag{32}$$

where $\underline{E}_t(\underline{I}_1)$ is the tangential electric field on the wire surface due to outgoing TEM currents. It is derived in Appendix A for the field due to outgoing TEM traveling currents in free space. In our case there is an infinite conducting plane near the wire. By the image theory, it is equivalent to the total field due to current on the wire and the current on its image, both in free space. Therefore, using (A-14), we obtain

$$E_x(\underline{I}_1) = \frac{-k}{2\pi\omega\varepsilon} \left(\frac{e^{-jkR_o}}{R_o} - \frac{e^{-jkR_o'}}{R_o'} \right) \tag{33}$$

where

$$R_o = \sqrt{a^2 + x^2} \tag{34}$$

$$R_o' = \sqrt{4d^2 + x^2} \tag{35}$$

Substituting (29) and (33) into (32), we find

$$Z_{11} = \frac{k}{2\pi\omega\varepsilon} \int_{-\infty}^{\infty} \left(\frac{e^{-jkR_o}}{R_o} - \frac{e^{-jkR'_o}}{R'_o}\right) e^{-jk|x|} dx \tag{36}$$

The integral can be evaluated as

$$\int_{-\infty}^{\infty} \frac{e^{-jkR_o}}{R_o} \cdot e^{-jk|x|} dx = \int_{0}^{\infty} \frac{e^{-jk(R_o+x)}}{R_o} dx + \int_{-\infty}^{0} \frac{e^{-jk(R_o-x)}}{R'_o} dx$$

$$= 2 \int_{0}^{\infty} \frac{e^{-jk(R_o+x)}}{R_o} dx = -\,2\,[Ci(ka) - jSi(ka)] \tag{37}$$

where

$$Ci(x) = -\int_{x}^{\infty} \frac{\cos u}{u} du \tag{38}$$

$$Si(x) = \int_{0}^{x} \frac{\sin u}{u} du \tag{39}$$

Using (37), we can reduce (36) to

$$Z_{11} = \frac{k}{\pi\omega\varepsilon} [Ci(2kd) - Ci(ka) + jSi(ka) - jSi(2kd)] \tag{40}$$

For $m \neq 1$, $n = 1$, substituting (30) and (33) into (22), we obtain

$$Z_{m1} = \frac{k}{2\pi\omega\varepsilon} \int_{P_{2m-1}}^{P_{2m+3}} T_m(x) \left(\frac{e^{-jkR_o}}{R_o} - \frac{e^{-jkR'_o}}{R'_o}\right) dx \tag{41}$$

where the notation is the same as that used in Appendix B. We use the same approximation as that in Appendix B, that is, the testing function $T_m(x)$ is approximated by four impulses. Hence, (41) becomes

$$Z_{m1} = \frac{k}{2\pi\omega\varepsilon} \sum_{A=1}^{4} C_m^x(A) \int_{\Delta X_{2m-2+A}} \left(\frac{e^{-jkR_o}}{R_o} - \frac{e^{-jkR'_o}}{R'_o}\right) dx \tag{42}$$

Using Green's function ψ as defined in (B-21), we can rewrite (42) as

$$Z_{ml} = \frac{2k}{\omega\varepsilon} \sum_{A=1}^{4} C_m^x(A) \; [\psi(0, Q_{2m-2+A}) - \psi(0, Q'_{2m-2+A})] \cdot \Delta X_{2m-2+A} \tag{43}$$

All the elements of [Z] matrix are now evaluated explicitly in (31), (40), (43), and (27).

(B) Evaluation of $[Y^a]$ and $[Y^b]$

According to the definition of $[Y^a]$ and $[Y^b]$, Eqs. (14) and (15), $[Y^a]$ and $[Y^b]$ are the matrices obtained by using the expansion function $\underline{M}_n$ as a magnetic current radiating into region a and b, respectively, with the infinite conducting plane present and the slot closed and the wire not present. Since the two regions a and b are identical half spaces, their generalized admittance matrices are same. Therefore we have

$$[Y^a] = [Y^b] = [Y^{hs}] \tag{44}$$

where $[Y^{hs}]$ is the generalized admittance matrix for the slot opening into half free space. Applying the image theorem, we find that $[Y^{hs}]$ is the admittance matrix obtained by using $2\underline{M}_n$ as magnetic current radiating into free space everywhere.

Now we specify $[Y^{hs}]$ for our narrow slot case. In this problem the width W is small relative to the wavelength and the length is large compared to its width. The electric field in the slot is principally transverse to the longer slot dimension (transverse to y), and has a known transverse variation given in the literature [7]. This transverse variation of electric field, or its equivalent magnetic current, is

$$f(x) = \frac{1}{\pi}\,[(\frac{w}{2})^2 - x^2]^{-1/2} \tag{44}$$

where w is the width of the slot. Also, the magnetic current has only a y component. Hence, the magnetic current expansion functions may be expressed as

$$\underline{M}_n(x,y) = f(x)\ \underline{T}_n(y) \qquad n = 1,2,\ldots,NA \tag{45}$$

where vector functions $\underline{T}_n(y)$ are triangle functions defined in Appendix B.

Now we evaluate the magnetic field $\underline{H}_t^a(\underline{M}_n)$ on the slot. For $\underline{H}_t$, we have the following formulation:

$$H_y(\underline{M}_n) = -\ j\omega F_y - \frac{\partial\phi_m}{\partial y} \tag{46}$$

$$F_y = \frac{\varepsilon}{4\pi}\iint_A M_n\,\frac{e^{-jkR}}{R}\,ds' \tag{47}$$

$$\phi_m = \frac{1}{4\pi\mu}\iint_A \sigma_m\,\frac{e^{-jkR}}{R}\,ds' \tag{48}$$

$$\sigma_m = -\frac{1}{j\omega}\,\nabla\cdot\underline{M}_n \tag{49}$$

where F_y is the only component of electric vector potential, and ϕ_m is the magnetic scalar potential. The integral is carried over the region where $\underline{M}_n$ exists. R is the distance from the field point, which is on the slot, to ds',

$$R = \sqrt{x'^2 + (y - y')^2}$$

Substituting this into (44) we calculate the integral S as

$$S = \iint_A M_n \frac{e^{-jkR}}{R}\, ds' = \int_{-\frac{w}{2}}^{\frac{w}{2}} dx' \int_{-\frac{b}{2}}^{\frac{b}{2}} \frac{\frac{1}{\pi}}{\sqrt{(\frac{w}{2})^2 - x'^2}} \frac{T_m(y') e^{-jk\sqrt{x'^2+(y-y')^2}}}{\sqrt{x'^2 + (y-y')^2}}\, dy'$$

$$= \int_{-\frac{b}{2}}^{\frac{b}{2}} T_m(y') \int_{-\frac{w}{2}}^{\frac{w}{2}} \frac{\frac{1}{\pi}}{\sqrt{(\frac{w}{2})^2 - x'^2}} \frac{e^{-jk\sqrt{x'^2+(y-y')^2}}}{\sqrt{x'^2 + (y-y')^2}}\, dx' dy' \tag{51}$$

The change of integration variable

$$x' = \frac{w}{2} \sin \frac{\alpha}{2}$$

reduces (51) to

$$S = \int_{-\frac{b}{2}}^{\frac{b}{2}} T_m(y') dy' \int_{-\pi}^{\pi} \frac{e^{-jk\sqrt{(y-y')^2 + \frac{w^2}{4} \sin^2 \frac{\alpha}{2}}}}{2\pi \sqrt{(y-y')^2 + \frac{w^2}{4} \sin^2 \frac{\alpha}{2}}}\, d\alpha$$

$$= \int_{-\frac{b}{2}}^{\frac{b}{2}} T_m(y') \frac{e^{-jk\sqrt{(y-y')^2 + \frac{w^2}{4}}}}{\sqrt{(y-y')^2 + \frac{w^2}{4}}}\, dy' \approx \int_{-\frac{b}{2}}^{\frac{b}{2}} T_m(y') \frac{e^{-jkR_e}}{R_e}\, dy' \tag{52}$$

Here we have used the relation

$$\frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{e^{-jk\sqrt{(y-y')^2 + \frac{w^2}{4} \sin^2 \frac{\alpha}{2}}}}{\sqrt{(y-y')^2 + \frac{w^2}{4} \sin^2 \frac{\alpha}{2}}}\, d\alpha \approx \frac{e^{-jk\sqrt{(y-y')^2 + (\frac{w}{4})^2}}}{\sqrt{(y-y')^2 + (\frac{w}{4})^2}} \tag{53}$$

and

$$R_e = \sqrt{(y - y')^2 + (\tfrac{w}{4})^2} \tag{54}$$

With this result, (47) through (49) can be reduced to

$$F_y = \frac{\varepsilon}{4\pi} \int_{-\frac{b}{2}}^{\frac{b}{2}} T_m(y') \frac{e^{-jkR_e}}{R_e} dy' \tag{55}$$

$$\phi_m = \frac{1}{4\pi\mu} \int_{-\frac{b}{2}}^{\frac{b}{2}} \sigma_m \frac{e^{-jkR_e}}{R_e} dy' \tag{56}$$

$$\sigma_m = -\frac{1}{j\omega} \frac{d}{dy'} T_m(y') \tag{57}$$

F_y of (46), together with (55) through (57), is recognized to be of the form as the magnetic vector potential associated with a thin wire of radius w/4. It is the dual problem to the formulation of [Z] matrix for a thin wire of radius w/4. Therefore, using this duality, we obtain the formula for the [Y] matrix of a slot with triangular expansion functions $T_m(y)$ to be

$$Y_{mn}^{hs} = 2 \left(j\omega\varepsilon\, Y_1 + \frac{1}{j\omega\mu} Y_2\right) \tag{58}$$

$$Y_1 = \sum_{A=1}^{4} \sum_{B=1}^{4} C_m^y(A)\, C_n^y(B)\, \Delta y_i \Delta y_j\, \psi(Q_i, Q_j) \tag{59}$$

$$Y_2 = \sum_{A=1}^{4} \sum_{B=1}^{4} D_m^y(A)\, D_n^y(B)\, \Delta y_i \Delta y_j\, \psi(Q_i, Q_j) \tag{60}$$

where

$$i = 2m - 2 + A \tag{61}$$

$$j = 2n - 2 + B \tag{62}$$

The Green's function ψ is defined similar to (B-21),

$$\psi(Q_i, Q_j) = \frac{1}{4\pi\Delta y_j} \int_{P_j}^{P_{j+1}} \frac{e^{-jkR_e}}{R_e} dy' \tag{63}$$

where Q_i is at the center of the ith segment Δy_i of the slot, P_j is the jth dividing point, and R_e is defined in (54). Note that the coefficient 2 in (58) comes from the fact that the magnetic field produced by $2\underline{M}_n$ in free space.

(C) Evaluation of [T] and [$\hat{T}$]

In order to evaluate [T] from (16), we have to calculate $\underline{H}_t^b(\underline{I}_n)$ on the slot. In our coordinate system, we have

$$\underline{H}_t^b(\underline{I}_n) = H_y(\underline{I}_n)\underline{u}_y$$

From the geometry shown in Fig. 3, we obtain the following relations:

$$H_y = \underline{u}_y \cdot H_\phi \, \underline{u}_\phi$$

$$\underline{u}_\phi = \cos\alpha \, \underline{u}_y + \sin\alpha \, \underline{u}_z$$

where $\underline{u}_y$ and $\underline{u}_z$ are unit vectors in the y and z directions, respectively, $\underline{u}_\phi$ is the unit azimuthal vector of the circle centered on the wire, and α

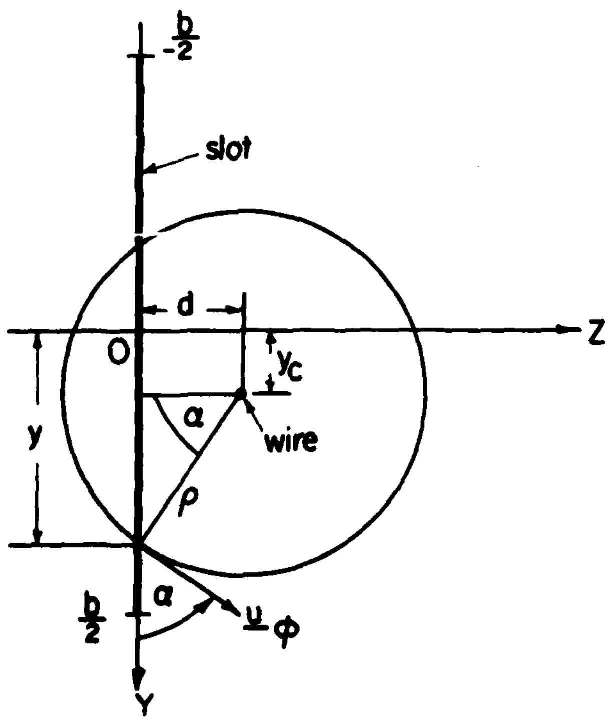


Fig. 3. Wire and slot in y-z plane.

is the angle between $\underline{u}_\phi$ and $\underline{u}_z$. It is given by

$$\cos\alpha = d/\rho \tag{64}$$

where

$$\rho = \sqrt{(y - y_c)^2 + d^2} \tag{65}$$

Therefore

$$H_y = \frac{d}{\sqrt{(y-y_c)^2 + d^2}} H_\phi \tag{66}$$

For n=1, $H_\phi = H_\phi(\underline{I}_1)$ is expressed in Appendix A, (A-13). Substitution into (66) gives

$$H_y(\underline{I}_1) = \frac{d}{\pi[(y-y_c)^2 + d^2]} e^{-jkR_o} \tag{67}$$

Note that (67) already includes the field due to the image of $\underline{I}_1$, and the wire is now taken in x direction instead of in the z direction as in Appendix A. Since the axis of the slot is in the y-z plane, we have $R_o = \rho$. Hence, (67) can be rewritten as

$$H_y(\underline{I}_1) = \frac{d}{\pi[(y-y_c)^2 + d^2]} e^{-jk\sqrt{(y-y_c)^2+d^2}} \tag{68}$$

Substitution of (23), (68) into (16) gives

$$\begin{aligned} T_{m1} &= \langle \underline{M}_m , \underline{H}_t(\underline{I}_1) \rangle_A \\ &= \frac{d}{\pi} \int_{-\frac{b}{2}}^{\frac{b}{2}} T_m(y) \frac{e^{-jk\sqrt{(y-y_c)^2+d^2}}}{(y-y_c)^2 + d^2} dy \end{aligned}$$

Also, $T_m(y)$ is approximated by four impulses as described in Appendix B.

This gives

$$T_{m1} = \frac{d}{\pi} \sum_{A=1}^{4} C_m^y(A) \frac{e^{-jk\sqrt{(y_{2m-2+A}-y_c)^2+d^2}}}{(y_{2m-2+A}-y_c)^2 + d^2} \Delta y_{2m-2+A} \qquad (69)$$

where $C_m^y(A)$ is defined similarly to (B-11) through (B-13), y_{2m-2+A} is at the center of the (2m-2+A)th segment of the slot, and Δy_{2m-2+A} is the length of the (2m-2+A)th segment.

For $n \neq 1$, we first calculate $H_y(\underline{I}_n)$. Using (30) for $\underline{I}_n$, we find that the vector potential has only an x component

$$A_x = \frac{\mu}{4\pi} \int_{\text{wire}} T_n(x') \frac{e^{-jkR}}{R} dx'$$

where R is the distance from the field point (on the axis of the slot) to dx'.

$$R = \sqrt{(x-x')^2 + (y-y_c)^2 + (z-d)^2} \qquad (70)$$

Hence,

$$H_y(\underline{I}_n^w) = \frac{1}{\mu} (\nabla \times \underline{A})_y = \frac{1}{\mu} \frac{\partial A_x}{\partial z}$$

$$= \frac{1}{4\pi} \int_{\text{wire}} T_n(x') (\frac{\partial}{\partial z} \frac{e^{-jkR}}{R}) dx'$$

where $\underline{I}_n^w$ is the current on the wire. The total field should include the field due to the image, $\underline{I}_n^i$, that is,

$$H_y(\underline{I}_n) = H_y(\underline{I}_n^w + \underline{I}_n^i) = H_y(\underline{I}_n^w) + H_y(\underline{I}_n^i) = 2H_y(\underline{I}_n^w)$$

$$= \frac{1}{2\pi} \int_{\text{wire}} T_n(x') (\frac{\partial}{\partial z} \frac{e^{-jkR}}{R}) dx'$$

$$= \frac{1}{2\pi} \int_{\text{wire}} T_n(x') [\frac{-(z-d)}{R^3} - \frac{jk(z-d)}{R^2}] e^{-jkR} dx'$$

on the axis of the slot (x = 0, z = 0). Therefore

$$H_y(\underline{I}_n) = \frac{1}{2\pi} \int_{wire} T_n(x') \left(\frac{d}{R^3} + \frac{jkd}{R^2}\right) e^{-jkR} dx' \tag{71}$$

Substitution of (23) and (71) into (16) gives

$$T_{mn} = \langle \underline{M}_m, H_t(\underline{I}_n) \rangle_A$$

$$= \frac{d}{2\pi} \int_{-\frac{b}{2}}^{\frac{b}{2}} T_m(y') \int_{wire} T_n(x') \left(\frac{1}{R^3} + \frac{jk}{R^2}\right) e^{-jkR} dx'dy'$$

Again $T_m(y)$ and $T_n(x)$ are approximated by four impulses. The result is

$$T_{mn} = \frac{d}{2\pi} \sum_{A=1}^{4} \sum_{B=1}^{4} C_m^y(A)\ C_n^x(B) \left(\frac{1}{R_{mnAB}^3} + \frac{jk}{R_{mnAB}^2}\right) e^{-jkR_{mnAB}} \Delta x_{2n-2+B} \Delta y_{2m-2+A}$$

$$(m = 1,2,\ldots NA,\ n = 2,3,\ldots NB) \tag{72}$$

where

$$R_{mnAB} = \sqrt{x_{2n-2+B}^2 + (y_{2m-2+A} - y_c)^2 + d^2} \tag{73}$$

is the distance from the center of the (2n-2+B)th segment on the wire to the center of the (2m-2+A)th segment on the slot. From (28) and (72), all the elements of $[\hat{T}]$ also can be evaluated.

(D) Evaluation of $\vec{I}^j$

The incident field considered here is a plane wave, which is a good approximation for field radiated by a source distant from the slot.

The propagation vector $\underline{k}$ is assumed to lie in the x-z plane, as shown in Fig. 4. There are two polarizations for the incident plane wave. One is vertical polarization for which the electric field is perpendicular to the incident plane, or in the y direction. The other is horizontal polarization for which the electric field is in the incident plane, or the magnetic field is in the y direction. In our narrow slot case, we may neglect the vertical polarization because it has no y component of magnetic field, which is the only component that couples to region b. Hence, we only need consider horizontal polarization.

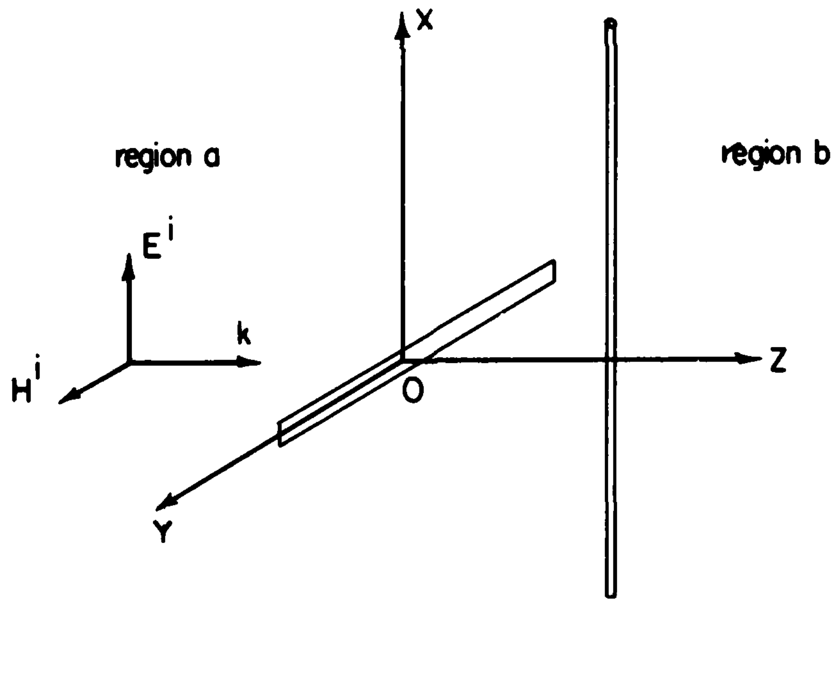


Fig. 4. The incident wave from region a.

For simplicity, we assume that a unit incident plane wave propagates in the z-direction, and is of the form,

$$\underline{H}^i = \underline{u}_y \, e^{-jkz} \tag{74}$$

$$\underline{E}^i = \underline{u}_x \, \eta \, e^{-jkz} \tag{75}$$

where η denotes the intrinsic impedance of free space. Therefore the tangential magnetic field H^i of incident wave on the slot is

$$\underline{H}^i \Big|_{\text{on slot}} = \underline{u}_y \, 2e^{-jkz} \Big|_{z=0} = 2\underline{u}_y \tag{76}$$

since the slot is closed by a conductor, forming a complete plane.

Substitution of (76) into (17) and (23) gives

$$\begin{aligned} I_m^i &= \langle \underline{M}_m, \underline{H}^i \rangle \\ &= 2 \int T_m(y)dy \\ &= 2\sum_{A=1}^{4} C_m^y(A) \; \Delta y_{2m-2+A} \end{aligned} \tag{77}$$

where $C_m^y(A)$ and Δy_{2m-2+A} are defined as before.

IV. SOLUTION TO EQUATIONS (13) AND (20)

So far, we have evaluated all the matrices in Equations (13) and (20) for our slot-wire problem. The computer programs are listed in Part 2 of this report. Before solving the slot-wire problem, we first reduce our problem to the dipole-wire problem. In this case we

solve (20) with $\vec{V}$ as known, and compare our moment solution with Kajfez's results. It is shown in Appendix C that these two solutions agree well with each other.

For slot-wire problem, given a set of geometric parameters, b, w, a, d and y_c, we can solve equations (13) and (20). We must also choose a definite region ($X < L/2$) on the wire for which the non-TEM modes are assumed to be significant. The solution $\vec{V}$ determines the axial distribution of magnetic current on the slot according to (8). The solution $\vec{I}$ determines the distribution of electric current on the wire according to (9). Here α_1 determines the amplitude of outgoing traveling TEM current on the wire, and α_2, α_3,... α_{NB} determine the evanescent currents on the wire. It is found that for the particular choice $d/\lambda < 0.1$ and $L/\lambda > 1.0$, a change in L/λ affects α_1 very little. However, there is a small change in the other α's. This indicates that the evanescent waves are localized to a region near the slot, and their values distant from the slot do not have a measurable effect on the field.

V. EQUIVALENT CIRCUIT

It is known that the equivalent circuit of a wire behind an aperture perforated conducting plane has the form as shown in Fig. 5. It applies to the TEM mode at the position x = 0. The equivalent sources

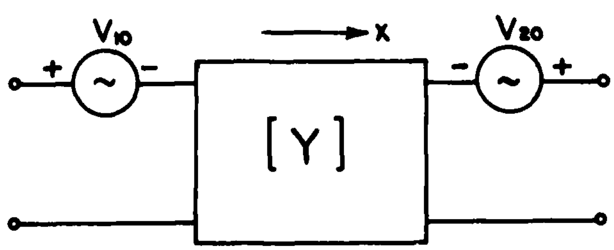


Fig. 5. Equivalent circuit.

V_{10} and V_{20} are due to the incident wave from region a. The two-port network parameters depend only on the geometry of the problem. For computational convenience, we will represent the network in terms of its [Y] parameters.

In order to determine the parameters of the two port network, we remove the incident wave from region a, apply a source to the transmission line formed by the wire and conducting plane, and then find the relationship between the port voltages and currents. To obtain the elements of Y parameters, we use the definition,

$$i_1 = T_{11}v_1 + Y_{12}v_2 \tag{78}$$

$$i_2 = Y_{21}v_1 + Y_{22}v_2 \tag{79}$$

where i_1, i_2, v_1 and v_2 are port currents and voltages at ports 1 and 2, respectively. The reference directions are as shown in Fig. 6.

To represent the excitation, we mathematically apply a TEM wave to the infinitely long transmission line. The incident fields are then

$$\underline{H}_{TEM} = I^+ \underline{h}\ e^{-jkx} \tag{80}$$

$$\underline{E}_{TEM} = V^+ \underline{e}\ e^{-jkx} \tag{81}$$

where I^+ and V^+ are the mode current and voltage, and $\underline{e}$ and $\underline{h}$ are

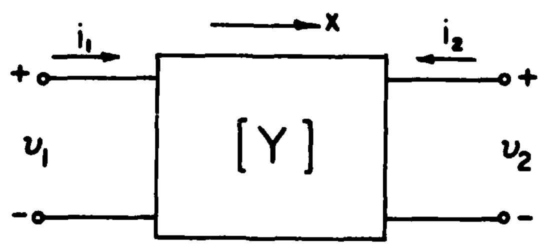


Fig. 6. Reference directions for i and v.

transverse electric and magnetic fields of TEM mode. These have the form [1]

$$\underline{e} = \frac{2d}{\pi} \frac{z(y-y_c)}{[(y-y_c)^2 + (z-d)^2]\,[(y-y_c)^2 + (z+d)^2]} \underline{u}_y$$

$$- \frac{d}{\pi} \frac{d^2 + (y-y_c)^2 - z^2}{[(y-y_c)^2 + (z-d)^2]\,[(y-y_c)^2 + (z+d)^2]} \underline{u}_z \quad (a << d) \qquad (82)$$

$$\underline{h} = \underline{u}_x \times \underline{e}$$

$$= \frac{2d}{\pi} \frac{z(y-y_c)}{[(y-y_c)^2 + (z-d)^2]\,[(y-y_c)^2 + (z+d)^2]} \underline{u}_z$$

$$+ \frac{d}{\pi} \frac{d^2 + (y-y_c)^2 - z^2}{[(y-y_c)^2 + (z-d)^2]\,[(y-y_c)^2 + (z+d)^2]} \underline{u}_y \qquad (83)$$

For the present case, in which the excitation is an incident TEM wave, every equation in Sec. II of this report is the same except that (2) must be changed to

$$\underline{H}^a = \underline{H}^a(\underline{M})$$

and (5) must be changed to

$$\underline{H}^b = \underline{H}^b(-\underline{M}) + \underline{H}^b(\underline{I}) + \underline{H}^i$$

where

$$\underline{H}^i = \underline{H}_{TEM} = I^+ \, \underline{h} \, e^{-jkx} \qquad (84)$$

the final equations are still (13) and (20), except for $\vec{I}^i$. Now $\vec{I}^i$ becomes

$$I_m^i = \langle -\underline{M}_m, \underline{H}_t^i \rangle$$

$$= - \int_{-\frac{b}{2}}^{\frac{b}{2}} T_m(y)\ \underline{u}_y \cdot I^+\ \underline{h}\ dy$$

$$= - \int_{-\frac{b}{2}}^{\frac{b}{2}} T_m(y)\ \frac{I^+}{\pi}\ \frac{d}{(y-y_c)^2 + d^2}\ dy$$

$$= - \frac{I^+}{\pi} \sum_{A=1}^{4} C_m^y(A)\ [\tan^{-1} (\frac{y_i^+ - y_c}{d}) - \tan^{-1} (\frac{y_i^- - y_c}{d})] \qquad (85)$$

where $C_m^y(A)$ are defined as before, and y_i^+ and y_i^- denote the two ends of the ith segment Δy_i $(i = 2m-2+A)$.

The incident TEM wave will excite both outgoing waves and evanescent waves. Because of the assumption of a narrow slot, and the symmetry of the problem, both outgoing waves have the same amplitude.

We now go through the procedures described in Sec. II once again. The only difference is that the excitation vector now is represented by (85) instead of (77). The others are all the same as before. We obtain the outgoing wave α_1 by solving (13) and (20).

We turn to the equivalent network in Fig. 6. The port voltage at port 1 is

$$v_1 = v^i - \alpha_1 Z_o \qquad (86)$$

where Z_o is the characteristic impedance of the transmission line formed by the conducting plane and the wire. v^i is the mode voltage which we applied to the transmission line traveling in the +x direction.

$$v^i = I^+ Z_o \tag{87}$$

The port current at port 1 is

$$v_1 = I^+ + \alpha_1 \tag{88}$$

At port 2, we have

$$v_2 = v^i + \alpha_1 Z_o \tag{89}$$

$$i_2 = -(I^+ + \alpha_1) \tag{90}$$

Because of symmetry, it follows that

$$Y_{11} = Y_{22}$$

From the reciprocity, it follows that

$$Y_{12} = Y_{21}$$

Hence, (78) and (79) can be reduced to

$$i_1 = Y_{11} v_1 + Y_{12} v_2 \tag{91}$$

$$i_2 = Y_{12} v_1 + Y_{11} v_2 \tag{92}$$

Solving (91) and (92) for Y_{11} and Y_{12}, we obtain

$$Y_{11} = (i_1 v_1 - i_2 v_2)/(v_1^2 - v_2^2) \tag{93}$$

$$Y_{12} = (i_2 v_1 - i_1 v_2)/(v_1^2 - v_2^2) \tag{94}$$

Substituting (86) through (90) into (93) and (94), we find

$$Y_{11} = -\frac{I^+ + \alpha_1}{2\alpha_1} Y_o \tag{95}$$

$$Y_{12} = \frac{I^+ + \alpha_1}{2\alpha_1} Y_o \tag{96}$$

where

$$Y_o = 1/Z_o \tag{97}$$

It is clear from (95) and (96) that

$$Y_{11} = -Y_{12} = Y \tag{98}$$

where

$$Y = -\frac{I^+ + \alpha_1}{2\alpha_1} Y_o$$

If I^+ equals unity, we have

$$Y = -\left(1 + \frac{1}{\alpha_1}\right) \frac{Y_o}{2} \tag{99}$$

Now, from (98), we can plot the equivalent network shown in Fig. 7. Finally, we evaluate the equivalent sources in Fig. 5 when the incident wave comes from region a. Since the network has the form of Fig. 7, we may reduce Fig. 5 to Fig. 8.

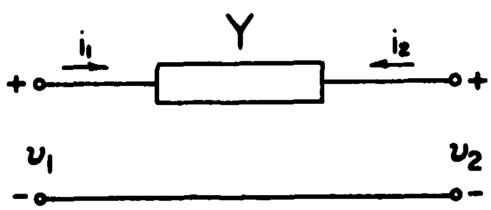


Fig. 7. Equivalent network at x = 0.

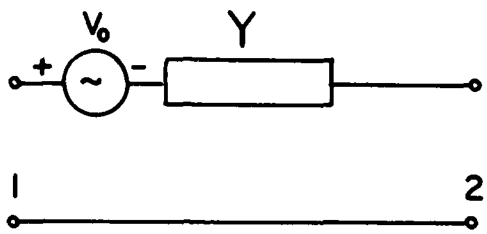


Fig. 8. Equivalent network for the original problem.

If both ports are terminated with matching loads, as shown in Fig. 9, the problem corresponds to that of an infinitely long wire. In this case, we can solve matrix equations (13) and (20) to obtain the outward traveling current on the wire.

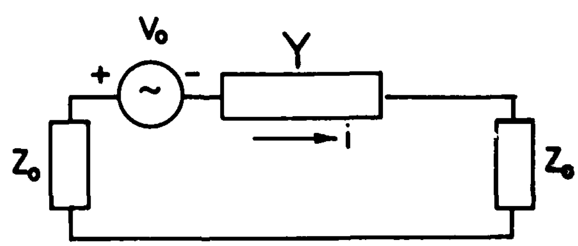


Fig. 9. Equivalent circuit for the calculation of V_o.

From Fig. 9, it is clear that

$$V_o = - I_o \left(\frac{1}{Y} + 2Z_o\right) \tag{100}$$

where I_o is the outgoing current when the wire is infinitely long and the wave comes from region a. The Y of the equivalent circuit is given by (99).

VI. NUMERICAL EXAMPLES AND DISCUSSION

A computer program was written to calculate the distribution of magnetic current on the slot and electric current on the wire for an infinite wire case. The parameters of the equivalent circuit are also calculated. This program is described and listed in Part 2. In this section, we give some numerical examples for a few cases of interest.

In Fig. 10, we show the distribution of the slot axial magnetic current along the slot axis for the typical case of $b = 0.5\lambda$, $w = 0.05\lambda$. The incident plane wave is incident normally on the slot from the region a. The infinitely long wire is located at the position $y_c = 0$ and $d = 0.1\lambda$. Also, the magnetic current for an isolated slot is plotted in Fig. 10. Note that the current for the isolated slot is different in magnitude from that when the wire is present.

In Fig. 11, we show the distribution of the total induced electric current on the wire. The real part is shown in Fig. 11a, and the imaginary part in Fig. 11b. In each case the total non-TEM current is also shown. Figure 10 shows that the field on the slot is influenced by the presence of the wire. Figure 11 shows that the induced current on the wire consists

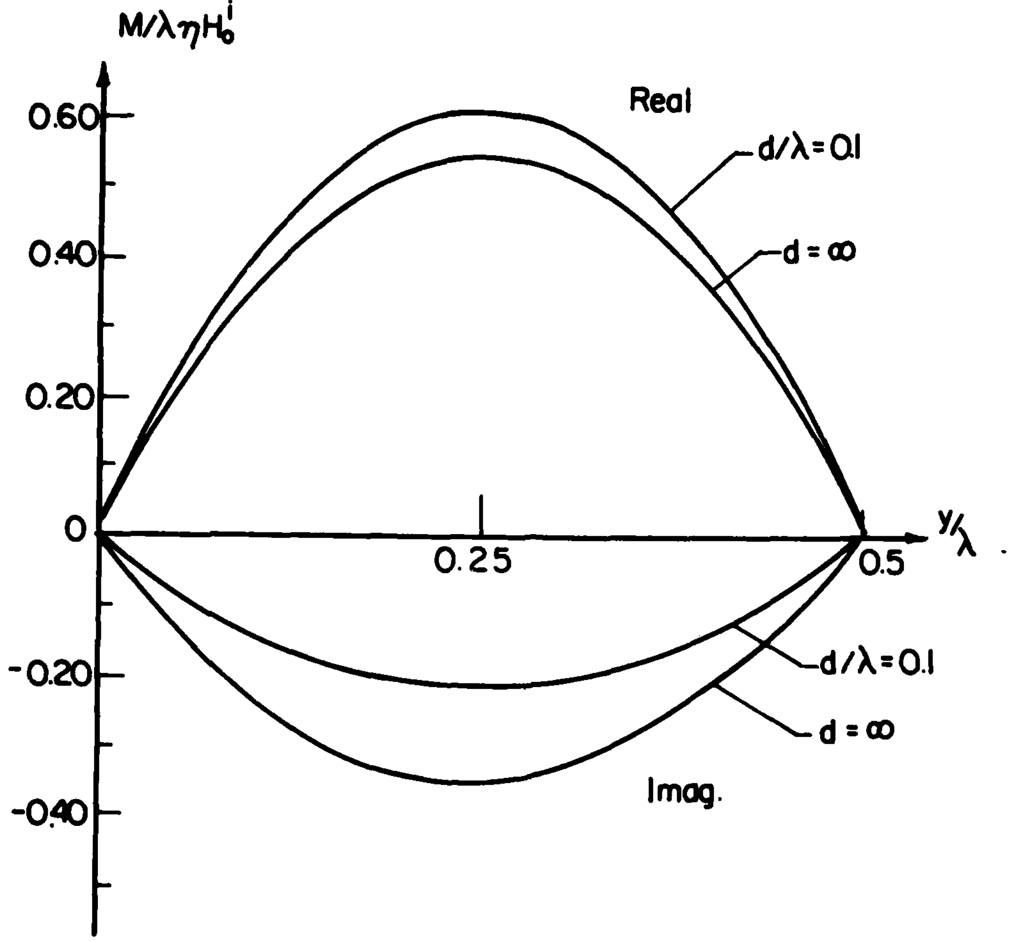


Fig. 10. Distribution of the axial slot magnetic current for $d/\lambda = 0.1$ and $d = \infty$ ($b/\lambda = 0.5$, $w/\lambda = 0.05$).

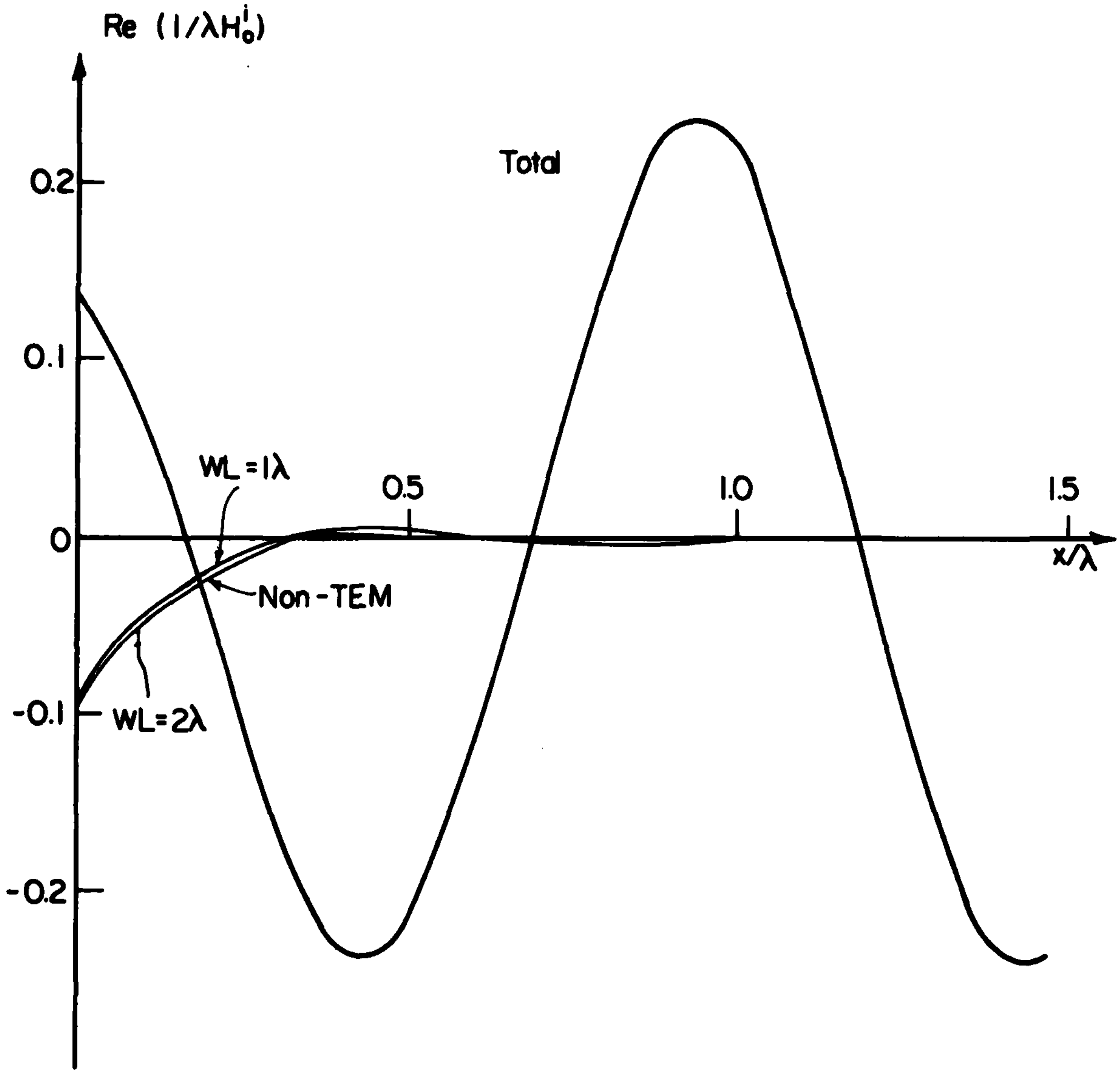


Fig. 11(a). The real part of current on the wire, for $b/\lambda = 0.5$, $w/\lambda = 0.05$, $a/\lambda = 0.001$, $d/\lambda = 0.1$.

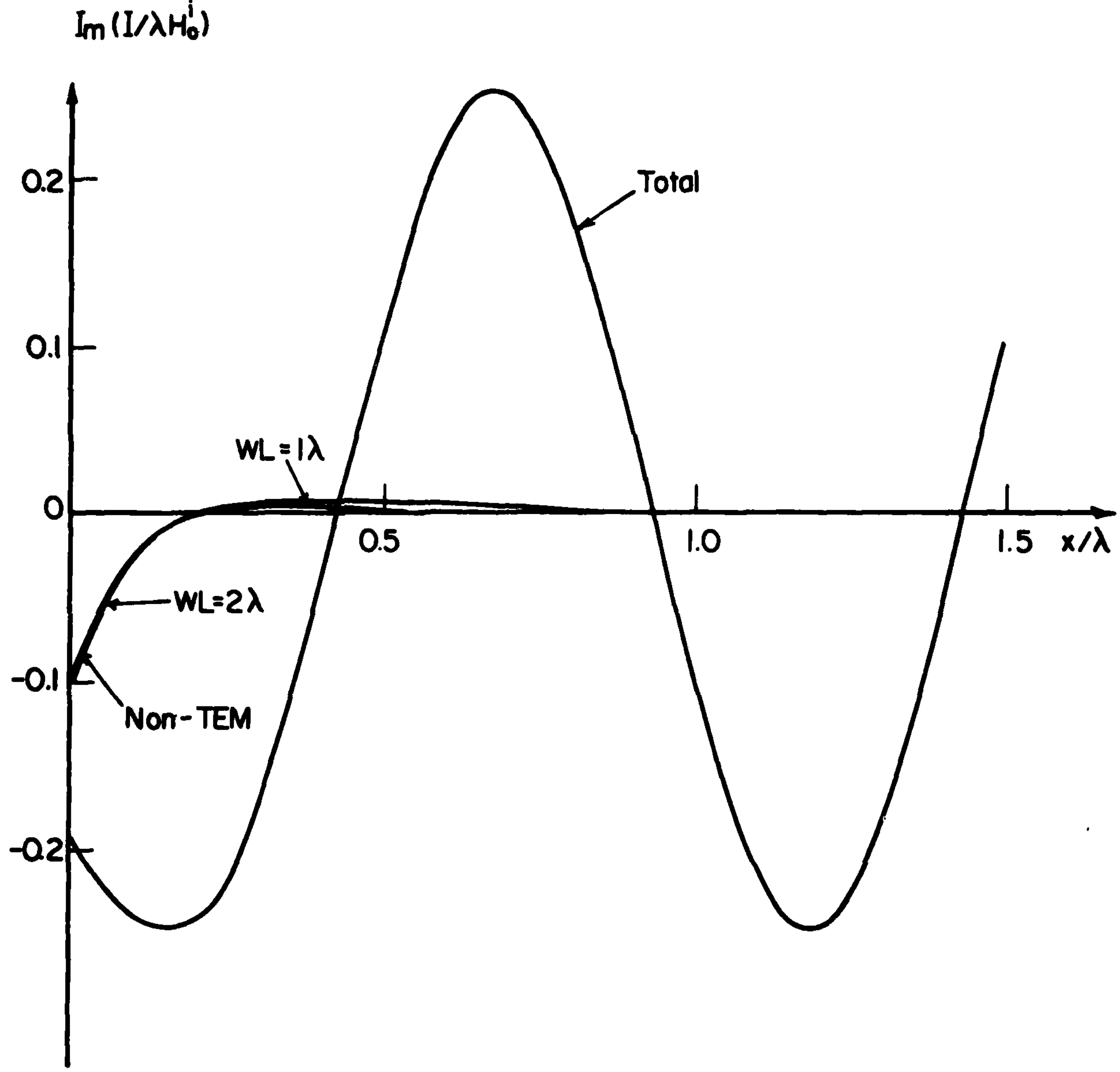


Fig. 11(b). The imaginary part of current on the wire, for $b/\lambda = 0.5$, $w/\lambda = 0.05$, $a/\lambda = 0.001$, $d/\lambda = 0.1$.

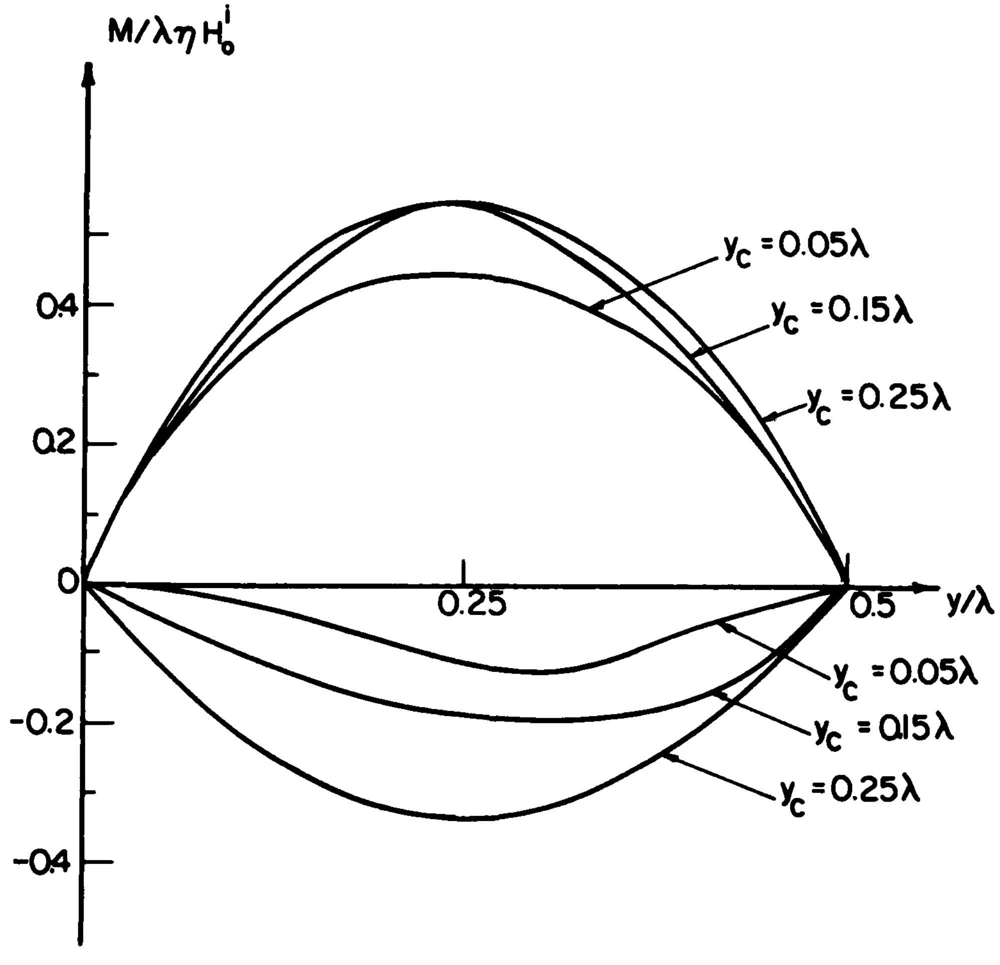


Fig. 12. Distribution of the axial slot magnetic current for various y_c. ($d/\lambda = 0.02$, $b/\lambda = 0.5$, $w/\lambda = 0.05$, $a/\lambda = 0.001$).

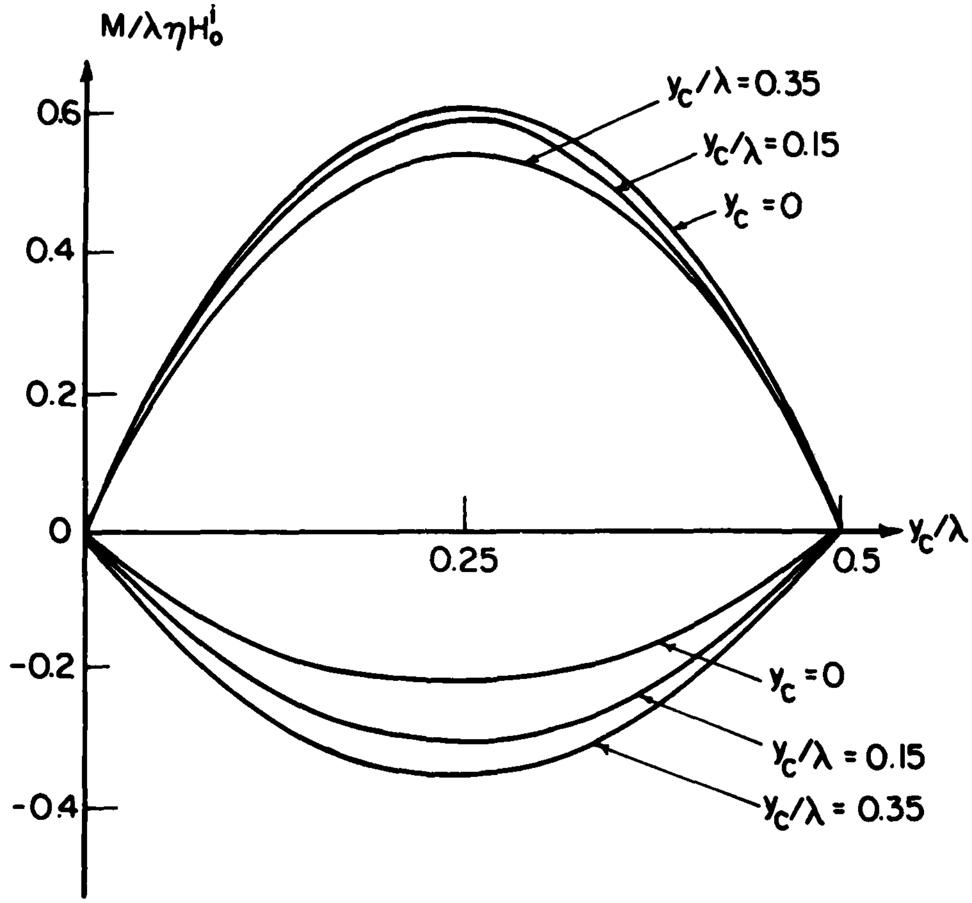


Fig. 13. Distribution of the axial slot magnetic current for various y_c/λ. ($d/\lambda = 0.1$, $b/\lambda = 0.5$, $w/\lambda = 0.05$, $a/\lambda = 0.001$).

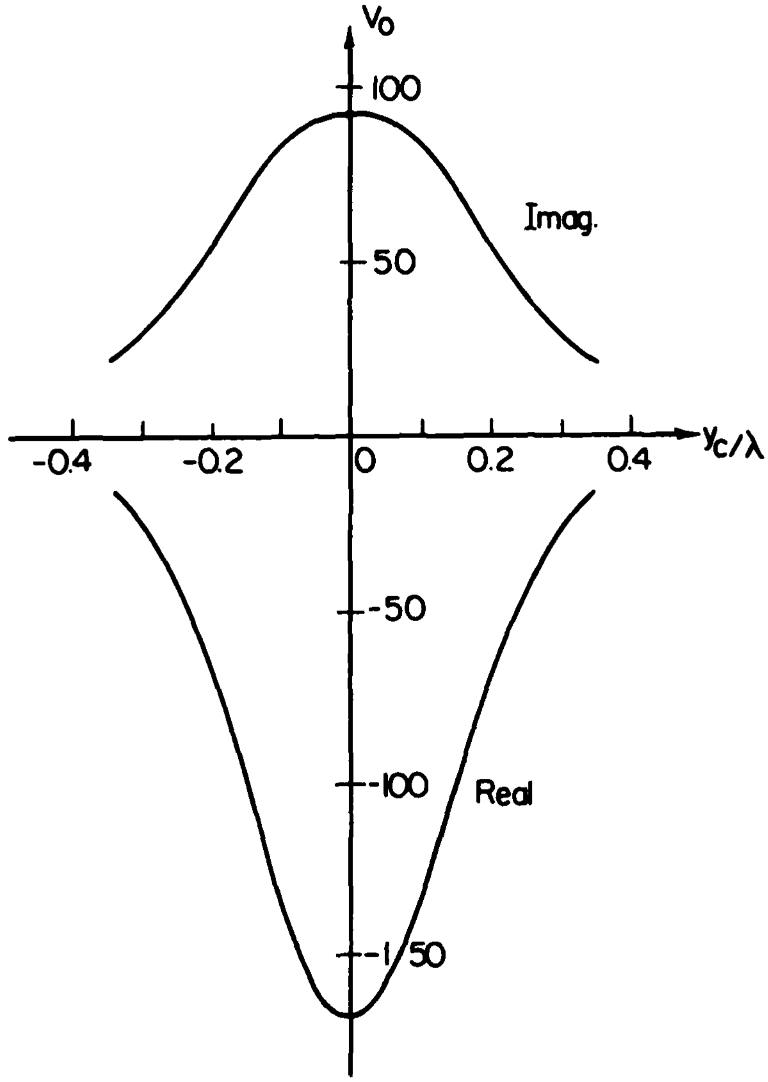


Fig. 14(a). Equivalent source for $d/\lambda = 0.1$, $b/\lambda = 0.5$, $w/\lambda = 0.05$, $a/\lambda = 0.001$. (λ = 1 meter, H_o^i = 1 Ampere/meter).

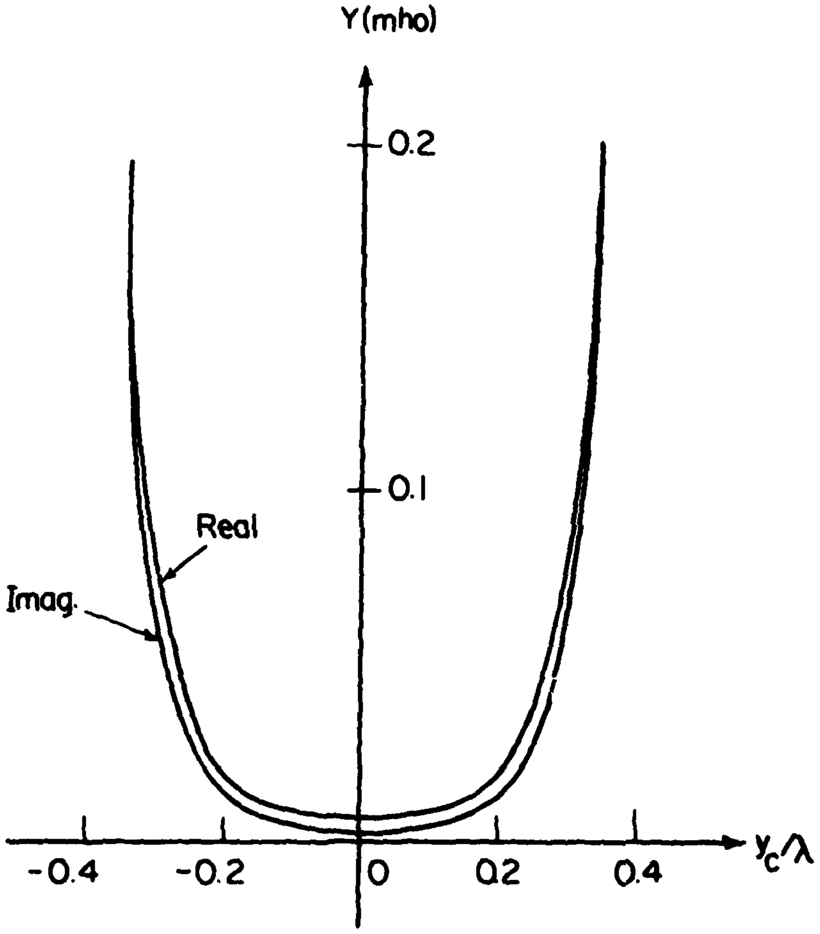


Fig. 14(b). Equivalent admittance for various y_c/λ. (d/λ = 0.1, b/λ = 0.5, w/λ = 0.05, a/λ = 0.001, λ = 1 meter, H_o^i = 1 Ampere/meter).

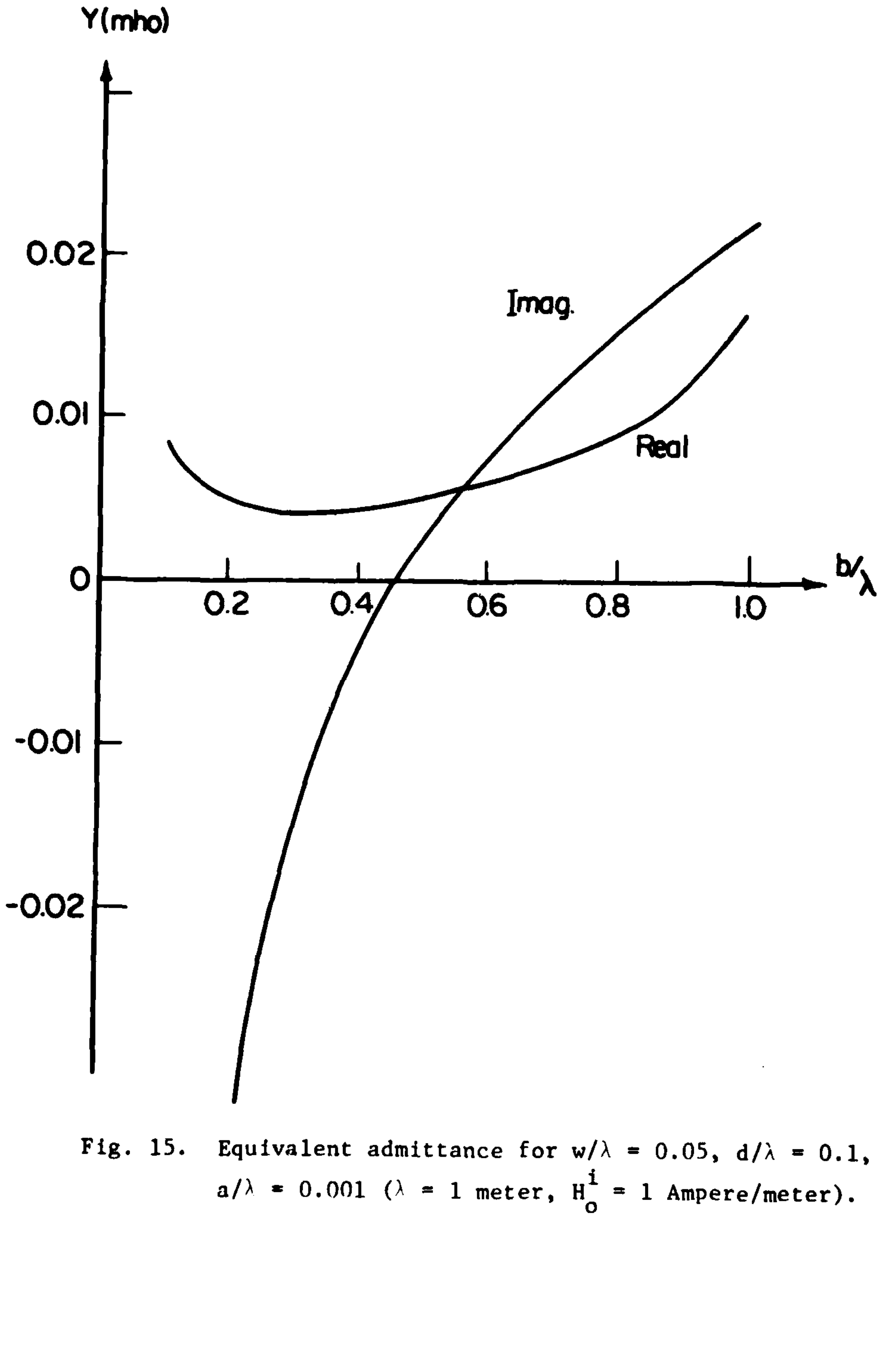


Fig. 15. Equivalent admittance for $w/\lambda = 0.05$, $d/\lambda = 0.1$, $a/\lambda = 0.001$ ($\lambda = 1$ meter, $H_o^i = 1$ Ampere/meter).

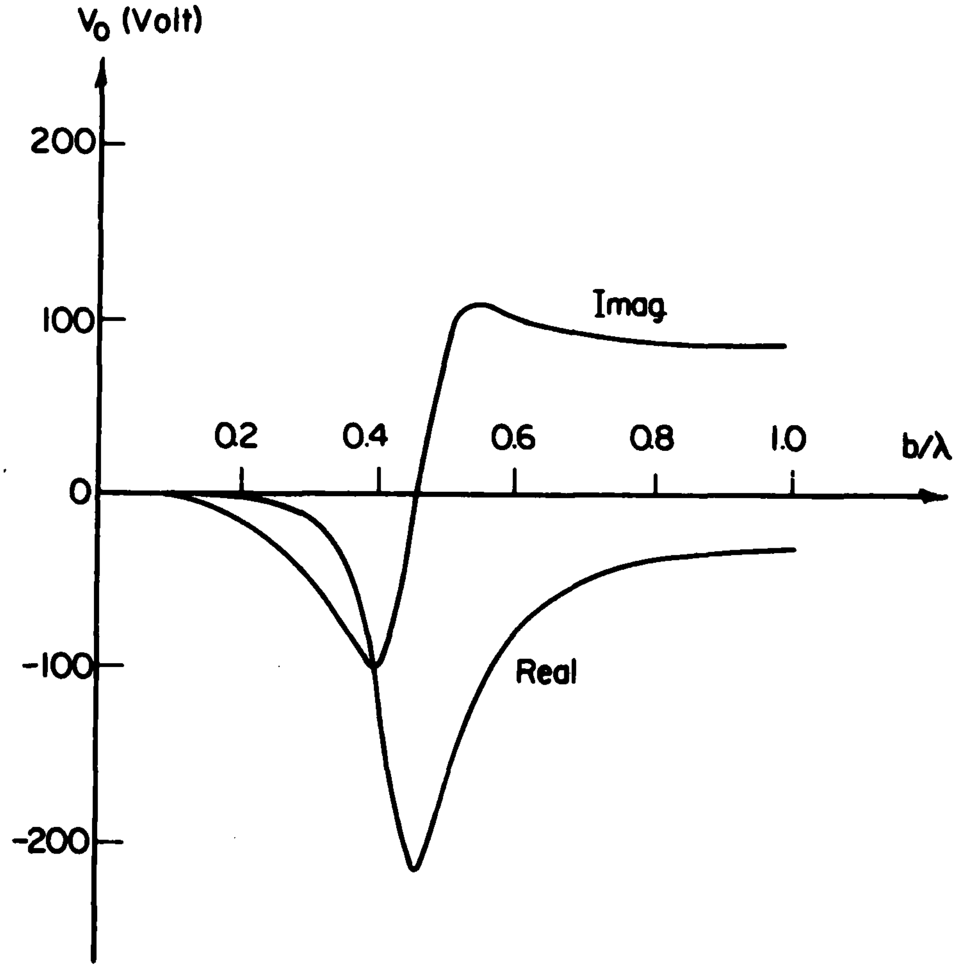


Fig. 16. Equivalent source for $w/\lambda = 0.05$, $d/\lambda = 0.1$, $a/\lambda = 0.001$ (λ = 1 meter, H_o^i = 1 Ampere/meter).

primarily of outgoing TEM mode currents. The higher order modes are significant near the slot, but they die out rapidly, as expected. The current move than one-half wavelength from the slot is essentially a single TEM mode. In Fig. 11, wL denotes the region on the wire where we assume that the higher order mode currents are considerable. Changing wL changes the total current hardly at all. Even for the higher order modes there is very little change as wL is changed.

In Fig. 12 and Fig. 13, we show how the position of the wire with respect to the center of the slot affects the distribution of magnetic current on the slot. Figure 12 is for the wire close to the conducting plane. Note that a displacement of the wire from the center of the slot changes both the symmetry and the magnitude of magnetic current. When the distance d is increased, it does not change the symmetry much, although the magnitude of magnetic current is still influenced by the displacement. This is shown in Fig. 13.

The variation of the equivalent network parameters with the displacement from the center of the slot is shown in Fig. 14 for $d/\lambda = 0.1$, $b/\lambda = 0.5$, $w/\lambda = 0.05$ and $a/\lambda = 0.001$. As expected, they are symmetrical about the centerpoint of the slot.

Figures 15 and 16 show the variation of the equivalent circuit parameters when the length of the slot is changed. For the case $w/\lambda = 0.05$, $d/\lambda = 0.1$, $a/\lambda = 0.001$, a resonance occurs in the vicinity of $b/\lambda = 0.46$ when the imaginary part of equivalent admittance vanishes, the equivalent source reaches its maximum. As expected, the equivalent admittance is inductive when $b/\lambda < 0.46$, and capacitive when $b/\lambda > 0.46$.

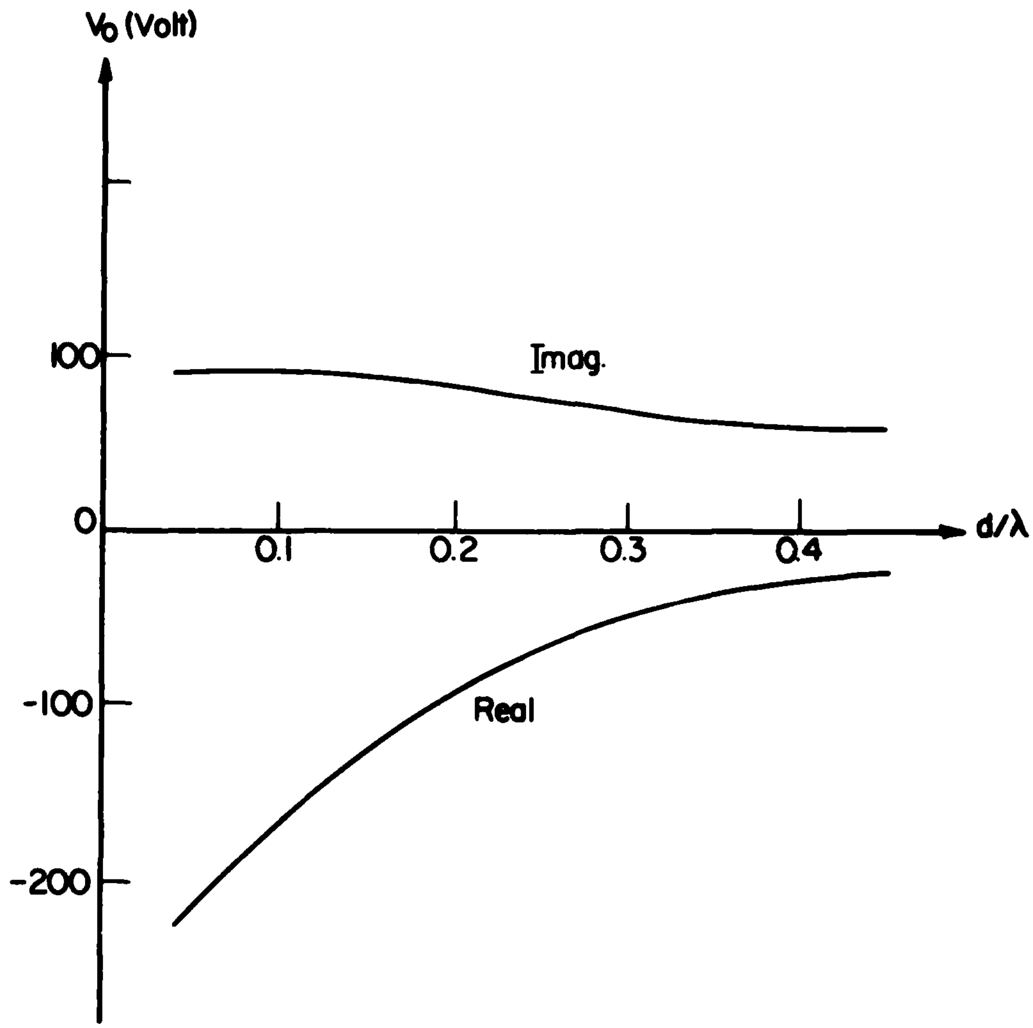


Fig. 17. Equivalent source for $w/\lambda = 0.05$, $b/\lambda = 0.5$, $a/\lambda = 0.001$. ($\lambda = 1$ meter, $H_o^i = 1$ Ampere/meter).

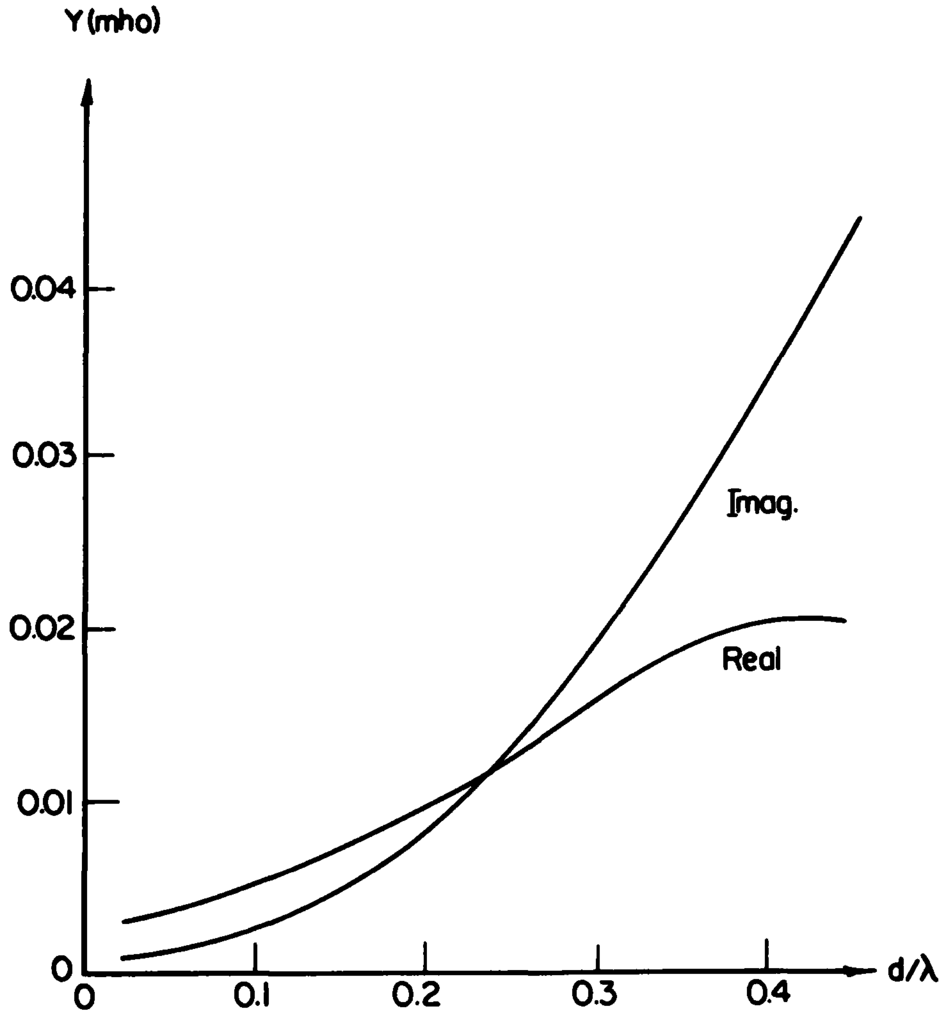


Fig. 18. Equivalent admittance for $b/\lambda = 0.5$, $w/\lambda = 0.05$, $a/\lambda = 0.001$. ($\lambda = 1$ meter, $H_o^i = 1$ Ampere/meter).

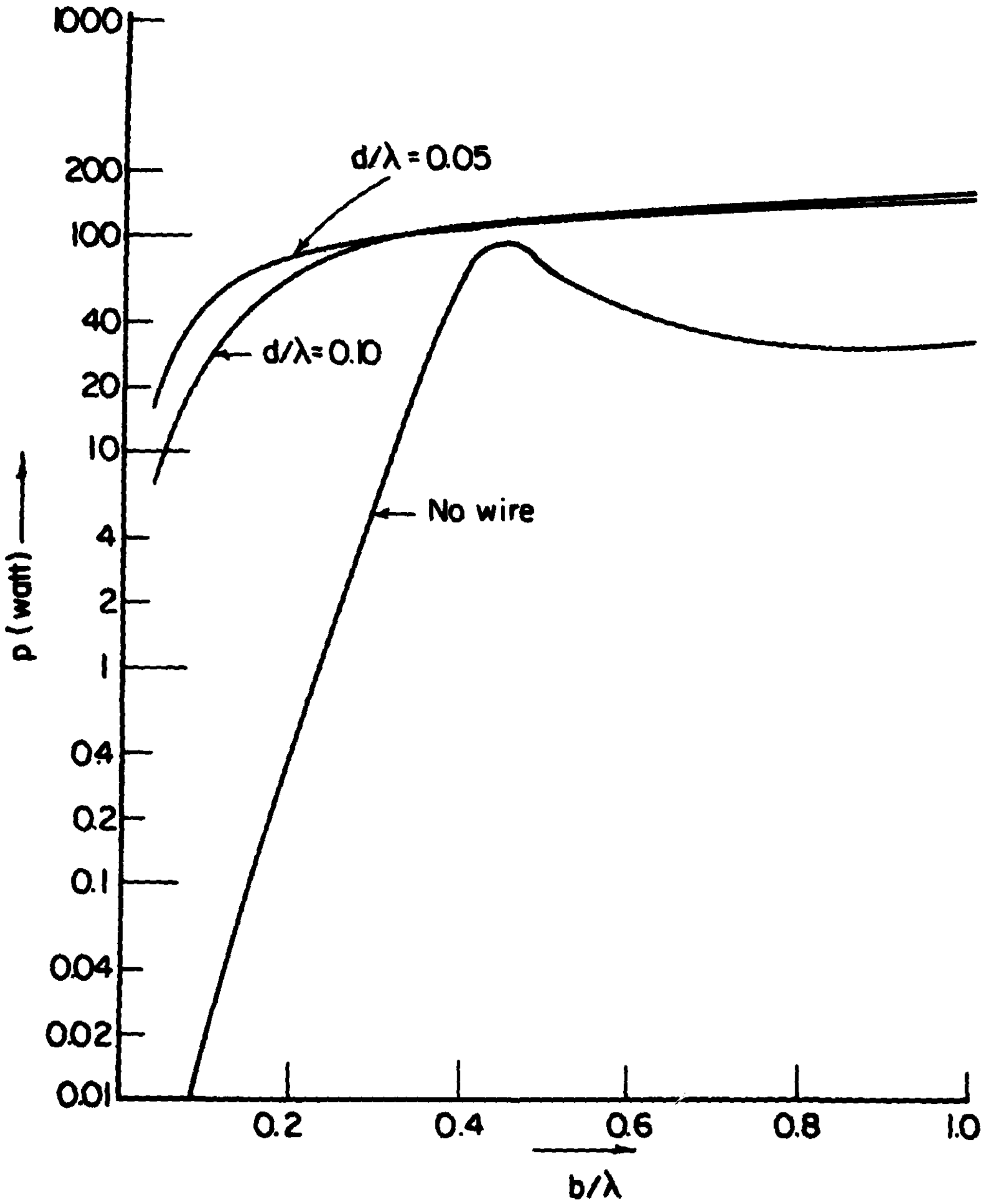


Fig. 19. Maximum power transferred to the load for $d/\lambda = 0.05$ and $d/\lambda = 0.10$, and power transmitted through the aperture when no wire is present.

Figures 17 and 18 show the variations of equivalent circuit parameters with distance d. It is evident from Figs. 17 and 18 that the magnitude of equivalent source decreases and the magnitude of equivalent admittance increases as the distance between the plane and the wire is increased.

It is interesting to compare the power transmitted from region a to region b through the slot when no wire is present to the maximum power coupled to the transmission line when the wire is present. For an isolated slot, the power transmitted from region a to region b is [6]

$$P_t = \tilde{\vec{V}}[Y^{hs}]^* \vec{V}^* \tag{101}$$

where $\vec{V}$ is the coefficient vector of the magnetic current on the isolated slot, and $\tilde{\vec{V}}$ is its transpose. $[Y^{hs}]$ is the generalized admittance matrix of the slot in half free space. The asterisks denote complex conjugate. Recalling $2[Y^{hs}]\vec{V} = \vec{I}^i$, where $\vec{I}^i$ is the excitation vector given by (77), we reduce (101) to

$$P_t = \frac{1}{2} \tilde{\vec{V}} \vec{I}^{i*} \tag{102}$$

Note that P_t is a complex power. The time-average power transmitted by the slot is

$$P_r = \mathrm{Re}(P_t) \tag{103}$$

where $\mathrm{Re}(P_t)$ denotes the real part of P_t.

The maximum power output in the transmission lines is obtained when it is terminated for maximum power transfer, as shown in Fig. 20. Here Z is the equivalent impedance

$$Z = 1/Y = R + jx \tag{104}$$

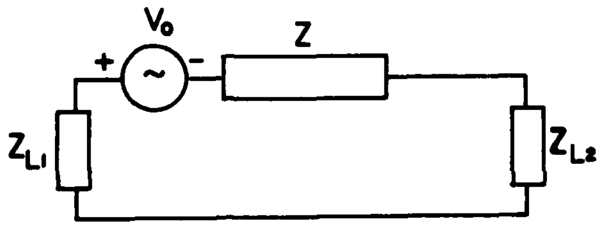


Fig. 20. The equivalent circuit for maximum power output

The loads, Z_{L1} and Z_{L2}, referred to x = 0, will receive maximum power when

$$Z_{L1} + Z_{L2} = R - jX \tag{105}$$

in which case the power is

$$P_L = \frac{|V_o|^2}{2R} \tag{106}$$

the results are plotted in Fig. 19. It can be seen that the power transmitted from region a to region b through the slot can be greatly increased by the presence of a nearby wire.

Appendix A

THE ELECTROMAGNETIC FIELD DUE TO TRAVELING WAVE CURRENTS ON AN INFINITELY LONG WIRE

To evaluate the [Z] and [T] matrices, it is necessary to know the electric and magnetic fields due to outward traveling currents on an infinitely long wire. The geometry and coordinates are shown in Fig. A-1. Here two traveling currents start from the origin and travel to infinity in the +z and -z directions. We are interested in the component of electric field parallel to the wire, E_z, and the component of magnetic field perpendicular to the wire, H_ϕ. So only these two components of the fields are evaluated. Because of symmetry, it is convenient to use a cylindrical coordinate system as shown in Fig. A-1. The point $p(\rho,\phi,z)$ is the field point where the fields are to be determined. For simplicity, we start

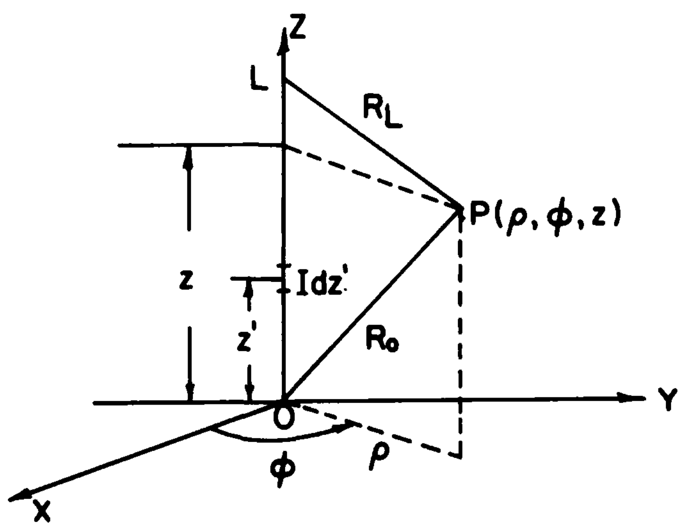


Fig. A-1. Geometry for fields due to a current along the z axis.

the derivation by assuming the wire be finite, of length L. From Fig. A-1 we see that the following relationships hold.

$$R = \sqrt{\rho^2 + (z - z')^2} \tag{A-1}$$

$$R_L = \sqrt{\rho^2 + (Z - L)^2} \tag{A-2}$$

$$R_o = \sqrt{\rho^2 + z^2} \tag{A-3}$$

The current is assumed to be traveling in the +Z direction on the wire, given by

$$\underline{I}^+ = C\ e^{-jkz}\ \underline{u}_z \qquad (z > 0) \tag{A-4}$$

where $\underline{u}_z$ is the unit vector in the z direction.

The expression for the vector potential at point P is

$$\underline{A} = A_z\,\underline{u}_z$$

$$A_z = \frac{\mu}{4\pi}\int_0^L C\ e^{-jkz'}\,\frac{e^{-jkR}}{R}\,dz'$$

$$= \frac{\mu C}{4\pi}\int_0^L \frac{e^{-jk(R+z')}}{R}\,dz' \tag{A-5}$$

The ϕ component of magnetic field at P is

$$H_\phi^+ = \frac{1}{\mu}\,(\nabla \times \underline{A})_\phi = -\frac{\partial A_z}{\partial \rho}$$

$$= -\frac{C}{4\pi}\int_0^L \frac{\partial}{\partial\rho}\left(\frac{e^{-jk(R+z')}}{R}\right)dz'$$

$$= -\frac{C}{4\pi}\int_0^L \left(\frac{-jk\rho}{R^2} - \frac{\rho}{R^3}\right)e^{-jk(R+z')}\,dz'$$

The integrand turns out to be a perfect differential. Integration therefore gives

$$H_\phi^+ = -\frac{C}{4\pi}\left[\frac{\rho e^{-jk(R+z')}}{R(R+z'-z)}\right]_{z'=0}^{z'=L}$$

$$= -\frac{C\rho}{4\pi}\left[\frac{e^{-jk(R_L+L)}}{R_L(R_L+L-z)} - \frac{e^{-jkR_o}}{R_o(R_o-z)}\right]$$

$$= -\frac{C\rho}{4\pi}\left[\frac{(R_L-L+z)e^{-jk(R_L+z)}}{R_L[R_L^2-(L-z)^2]} - \frac{(R_o+z)\,e^{-jkR_o}}{R_o(R_o^2-z^2)}\right]$$

From (A-2) and (A-3) we obtain

$$R_L^2 - (L-z)^2 = R_o^2 - z^2 = \rho^2$$

Hence,

$$H_\phi^+ = -\frac{C}{4\pi\rho}\left[\left(1-\frac{L-z}{R_L}\right)e^{-jk(R_L+L)} - \left(1+\frac{z}{R_o}\right)e^{-jkR_o}\right] \qquad \text{(A-6)}$$

Now let $L \to \infty$ and note that $\lim_{L\to\infty} \frac{L-z}{R_L} = 1$. Now (A-6) reduces to

$$H_\phi^+ = \frac{C}{4\pi\rho}\left(1+\frac{z}{R_o}\right)e^{-jkR_o} \qquad \text{(A-7)}$$

This is the ϕ component of magnetic field at point P due to a current traveling in the +Z direction, starting at $z = 0$.

The electric field can be obtained from the magnetic field by recalling that

$$\nabla \times \underline{H} = j\omega\varepsilon\underline{E}$$

from which we find

$$E_z^+ = \frac{1}{j\omega\varepsilon}\frac{1}{\rho}\left[\frac{\partial}{\partial\rho}(\rho H_\phi) - \frac{\partial H_\rho}{\partial\phi}\right]$$

Because of symmetry there is no variation in the ϕ direction. Therefore

$$E_z^+ = \frac{1}{j\omega\varepsilon}\frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho H_\phi) \qquad \text{(A-8)}$$

Substituting (A-7) into (A-8), we obtain

$$E_z^+ = -\frac{C}{4\pi j\omega\varepsilon}\left(\frac{jk}{R_o} + \frac{jkz}{R_o^2} + \frac{z}{R_o^3}\right) e^{-jkR_o} \qquad \text{(A-9)}$$

E_z^+ is the tangential component of electric field at point p due to a current traveling along the +z direction, starting at z = 0.

Now assume we have an outward traveling current starting at z = 0 and extending to $z = -\infty$,

$$\underline{I}^- = C\ e^{jkz}\ \underline{u}_{-z} \qquad (z < 0) \qquad \text{(A-10)}$$

To obtain its fields H_ϕ^- and E_z^-, all we need to do is to replace z by -z in (A-7) and (A-9). It follows that

$$H_\phi^- = \frac{C}{4\pi\rho}\left(1 - \frac{z}{R_o}\right) e^{-jkR_o} \qquad \text{(A-11)}$$

$$E_z^- = -\frac{C}{4\pi j\omega\varepsilon}\left(\frac{jk}{R_o} - \frac{jkz}{R_o^2} - \frac{z}{R_o^3}\right) e^{-jkR_o} \qquad \text{(A-12)}$$

The total field is the sum of those due to the +z traveling current and the -z traveling current, or

$$H_\phi = H_\phi^+ + H_\phi^- = \frac{C}{2\pi\rho} e^{-jkR_o} \qquad \text{(A-13)}$$

$$E_z = E_z^+ + E_z^- = -\frac{Ck}{2\pi\omega\varepsilon R_o} e^{-jkR_o} \qquad \text{(A-14)}$$

where

$$\rho = \sqrt{x^2 + y^2} \qquad \text{(A-15)}$$

and R_o is given by (A-3).

Appendix B

GENERALIZED Z-MATRIX OF A WIRE ABOVE AN INFINITE CONDUCTING PLANE

The problem is to evaluate the elements of the generalized Z-matrix for a finite thin wire above an infinite conducting plane. We follow the procedure used by Chao and Strait [8]. The geometry and coordinates are shown in Fig. B-1. For a thin wire, the following approximations are made:

1. The current is assumed to flow only in the axial direction of the wire.
2. The surface current is assumed to be circumferentially independent and accounted for by a total axial current I.
3. The only boundary condition is that the axial component of electric field be zero on the wire surface.

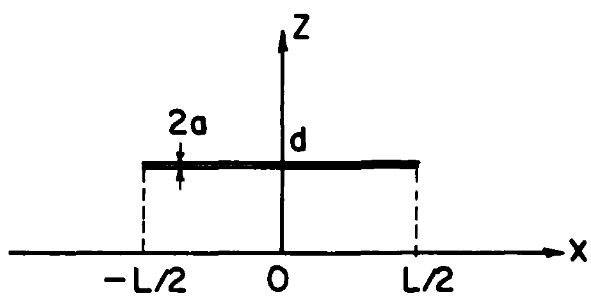


Fig. B-1. A wire above a conducting plane.

With these approximations in mind, we evaluate the [Z] matrix as follows. Recalling the definition of Z_{mn}, we start from

$$Z_{mn} = - \langle \hat{\underline{I}}_m, \underline{E}_t(\underline{I}_n) \rangle \tag{B-1}$$

where $\underline{I}_n$ is the nth expansion function of electric current on the wire, and $\underline{E}_t(\underline{I}_n)$ is the X component of electric field due to $\underline{I}_n$. Here we use Galerkin's method, so that the testing function $\hat{\underline{I}}_m$ is the same as expansion function $\underline{I}_m$.

For relatively fast convergence, we choose triangle functions as current expansion functions, that is,

$$\underline{I}_n(x) = \underline{T}_n(x)$$

The wire is divided into (2NB + 2) segments, where NB is the number of triangles as well as expansion functions. Each triangle has unit height with peak at the point p_{2n+1}, the direction of $\underline{T}_n(x)$ is coincident with the axis of the wire as shown in Fig. B-2. Each triangle function T_n

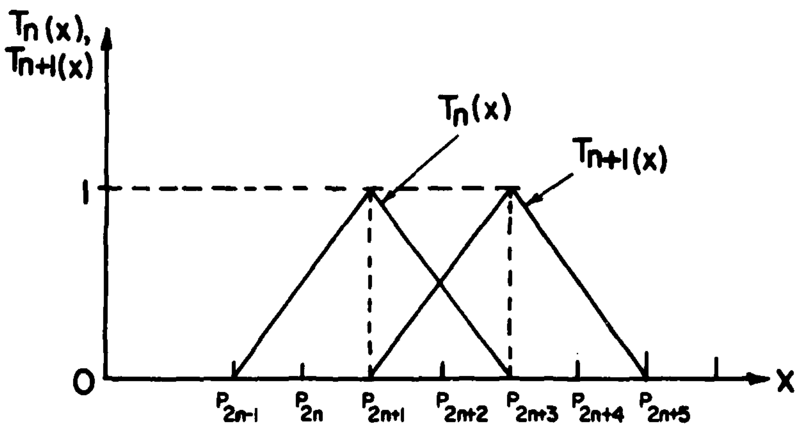


Fig. B-2. Triangle functions $T_n(x)$, $T_{n+1}(x)$.

is non-zero only over four segments. Successive triangles overlap two segments except at the ends of the wire.

Note that $\underline{E}_t(\underline{I}_n)$ is evaluated with the conducting plane present. From image theory [4], this is equivalent to the field produced in region z > 0 by $\underline{I}_n$ and its image $\underline{I}'_n$ without the conducting plane. Note that $\underline{I}'_n = -\underline{I}_n$ as shown in Fig. B-3. Therefore the x component of electric field on the surface of the wire can be expressed by

$$E_x = -j\omega A_{nx} - \frac{\partial \phi_n}{\partial x} \qquad \text{on the wire.} \tag{B-2}$$

Here A_{nx}, ϕ_n are the vector potential and the scalar potential due to $\underline{I}_n$ and its image $\underline{I}'_n$, given by

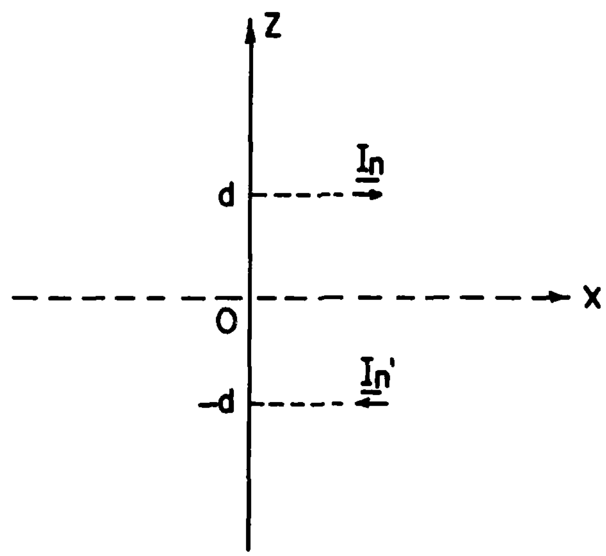


Fig. B-3. $\underline{I}_n$ and its image $\underline{I}'_n$.

$$A_{nx} = \frac{\mu}{4\pi} \int_{P_{2n-1}}^{P_{2n+3}} T_n(x') \left(\frac{e^{-jkR}}{R} - \frac{e^{-jkR'}}{R'}\right) dx' \tag{B-3}$$

$$\phi_n = \frac{1}{4\pi\varepsilon} \int_{P_{2n-1}}^{P_{2n+3}} \sigma_n(x') \left(\frac{e^{-jkR}}{R} - \frac{e^{-jkR'}}{R'}\right) dx' \tag{B-4}$$

$$\sigma_n = - \frac{1}{j\omega} \frac{d\, T_n(x')}{dx'} \tag{B-5}$$

where

$$R = \sqrt{(x - x')^2 + a^2} \tag{B-6}$$

$$R' = \sqrt{(x - x')^2 + (2d)^2} \tag{B-7}$$

are the distances from the current I_n and I_n' to the surface of the wire, respectively.

Substituting (B-2) into (B-1), we have

$$Z_{mn} = \int_{P_{2m-1}}^{P_{2m+3}} T_m(x) \left(j\omega A_{nx} + \frac{d}{dx} \phi_n\right) dx \tag{B-8}$$

From

$$\frac{d}{dx} (T_m \phi_n) = T_m \frac{d\phi_n}{dx} + \phi_n \frac{dT_m}{dx}$$

it follows that

$$\int_{P_{2m-1}}^{P_{2m+3}} d(T_m \phi_n) = \int_{P_{2m-1}}^{P_{2m+3}} T_m d\phi_n + \int_{P_{2m-1}}^{P_{2m+3}} \phi_n dT_m \tag{B-9}$$

The left hand side of (B-9) is zero, since T_m is zero at the ends, P_{2m-1} and P_{2m+3}. Hence (B-8) can be reduced to

$$Z_{mn} = \int_{P_{2m-1}}^{P_{2m+3}} dx \int_{P_{2n-1}}^{P_{2n+3}} [j\omega\mu\, T_m(x)T_n(x') + \frac{1}{j\omega\varepsilon} \frac{dT_m(x)}{dx} \frac{dT_n(x')}{dx'}](\frac{e^{-jkR}}{4\pi R} - \frac{e^{-jkR'}}{4\pi R'})dx' \tag{B-10}$$

In evaluating the integral of (B-10), triangle T_n is conveniently approximated by four pulses as shown in Fig. B-4, and $\frac{dT_n}{dx}$ is also represented by four pulses as shown in Fig. B-4. T_m and $\frac{dT_m}{dx}$ are approximated by four impulses as shown in Fig. B-5. The pulse amplitudes are

$$C_n(1) = \frac{\frac{1}{2}\Delta x_{2n-1}}{\Delta x_{2n-1} + \Delta x_{2n}} \tag{B-11}$$

$$C_n(2) = \frac{\Delta x_{2n-1} + \frac{1}{2}\Delta x_{2n}}{\Delta x_{2n-1} + \Delta x_{2n}} \tag{B-12}$$

$$C_n(3) = \frac{\frac{1}{2}\Delta x_{2n+1} + \Delta x_{2n+2}}{\Delta x_{2n+1} + \Delta x_{2n+2}} \tag{B-13}$$

$$C_n(4) = \frac{\frac{1}{2}\Delta x_{2n+2}}{\Delta x_{2n+1} + \Delta x_{2n+2}} \tag{B-14}$$

$$D_n(1) = D_n(2) = \frac{1}{\Delta x_{2n-1} + \Delta x_{2n}} \tag{B-15}$$

$$D_n(3) = D_n(4) = \frac{-1}{\Delta x_{2n+1} + \Delta x_{2n+2}} \tag{B-16}$$

The impulse amplitudes are $\Delta x_{2m-2+i}\, C_m(i)$, $\Delta x_{2m-2+i}\, D_m(i)$ $(i=1,2,3,4)$.

With these approximations, (B-10) reduces to

$$Z_{mn} = j\omega\mu Z_1 + \frac{1}{j\omega\varepsilon} Z_2 \tag{B-17}$$

where

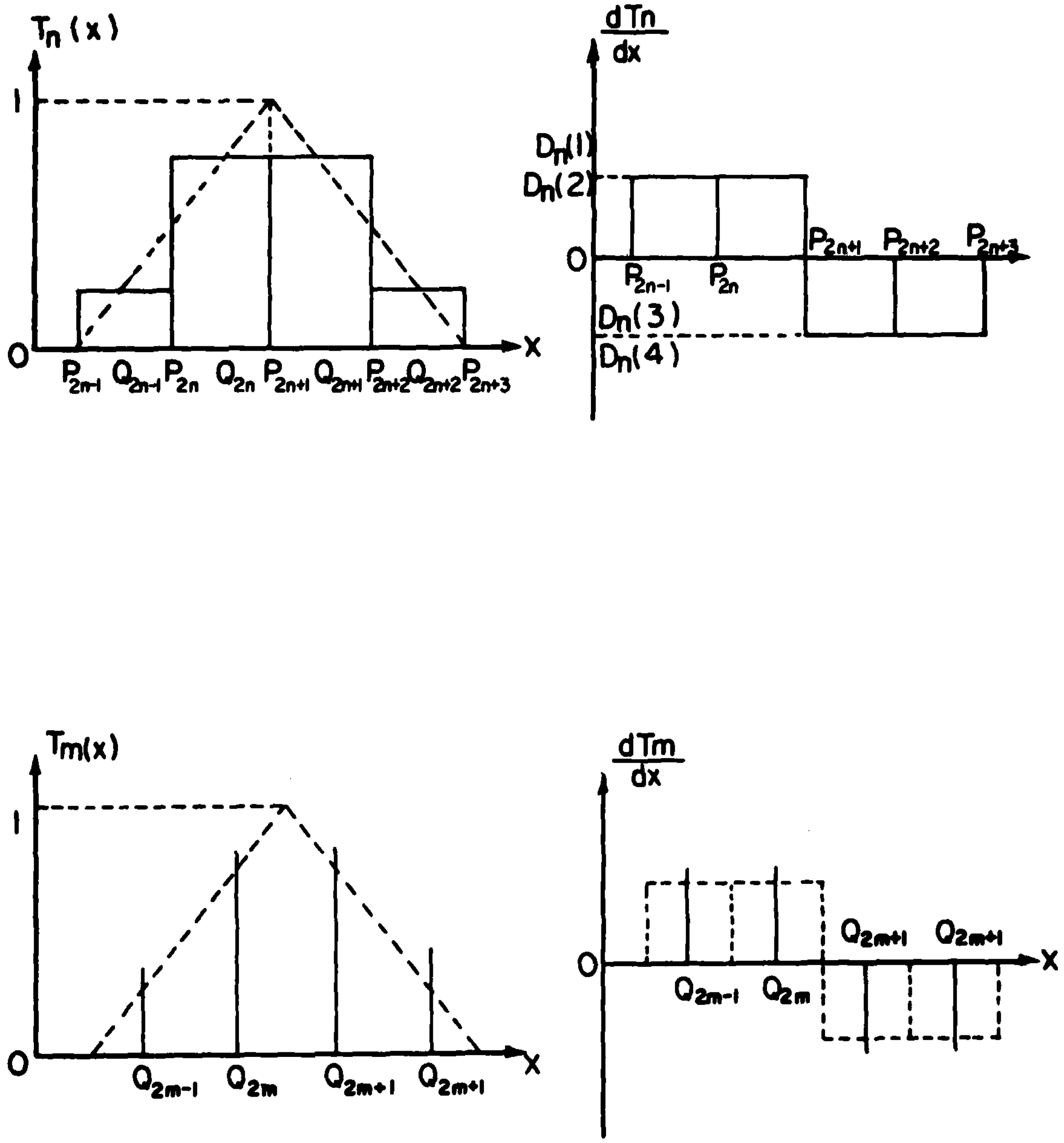


Fig. B-5. Approximations to $T_m(x)$ and $\frac{dT_m(x)}{dx}$

$$Z_1 = \sum_{A=1}^{4} \sum_{B=1}^{4} C_m(A)\ C_n(B)\ \Delta x_{2m-2+A}\ \Delta x_{2n-2+B}\ \Psi(Q_{2m-2+A},\ Q_{2n-2+B}) \qquad \text{(B-18)}$$

$$Z_2 = \sum_{A=1}^{4} \sum_{B=1}^{4} D_m(A)\ D_n(B)\ \Delta x_{2m-2+A}\ \Delta x_{2n-2+B}\ \Psi(Q_{2m-2+A},\ Q_{2n-2+B}) \qquad \text{(B-19)}$$

Here

$$\Psi(Q_i,\ Q_j) = \psi(Q_i,\ Q_j) - \psi(Q_i,\ Q_j') \qquad \text{(B-20)}$$

and Green's function ψ is defined as [10]

$$\psi(Q_i,\ Q_j) = \frac{1}{4\pi\Delta x_j} \int_{P_j}^{P_{j+1}} \frac{e^{-jkR}}{R}\, dx \qquad \text{(B-21)}$$

where Q_i is at the center of the ith segment of the wire, Q_j is at the center of the jth segment of the wire, Q_j' is at the point of the image of Q_j, and R is the distance from Q_i to dx.

Appendix C

APPLICATION OF EQUATIONS (13) AND (20) TO THE MAGNETIC DIPOLE AND WIRE PROBLEM

Figure C-1 shows the problem to be considered and defines the coordinates and notation to be used. An infinite conducting plane covers the entire $z = 0$ plane. An infinitely long thin wire is oriented in the direction of the x-axis at a distance $z=d$ from it. The source is a unit magnetic dipole situated at the origin. What we wish to find is the electric current on the wire. It is easy to solve this problem by the method of moments, as described in [5]. The problem is equivalent to a wire antenna scattering problem in which the source is an equivalent dipole in free space. The wire is the scatterer and

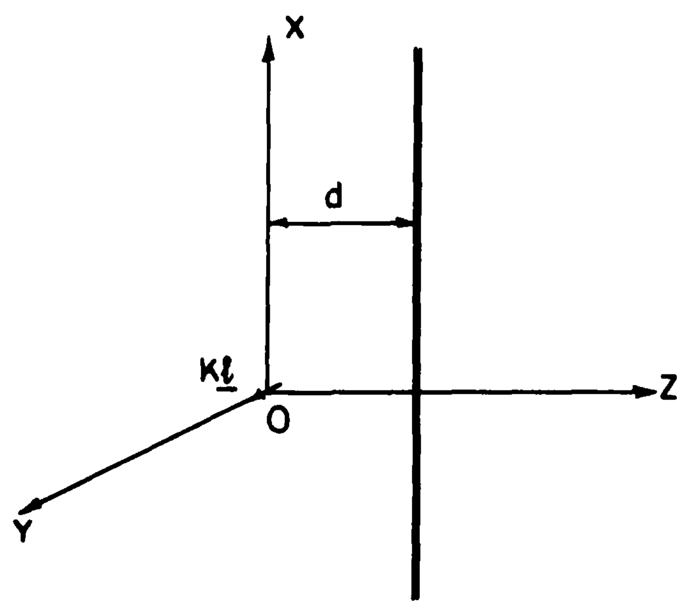


Fig. C-1. Dipole-wire problem.

the equivalent dipole when imaged in the conducting plane, is $2K\underline{\ell}$. Hence, we have to solve matrix equation

$$[Z]\ \vec{I} = \vec{V} \tag{C-1}$$

where $\vec{I}$ is generalized current to be determined, $\vec{V}$ is generalized voltage defined as

$$\vec{V} = \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix}, \quad V_n = \langle W_n, E^i \rangle \tag{C-2}$$

where W_n is the nth testing function on the wire. E^i is the tangential electric field, E_x^i, due to $2K\underline{\ell}$. Finally, [Z] is the generalized impedance matrix of the wire.

The same analysis as used in Section II(A) applies. Hence, we may use the same expansion and testing functions as previously, giving the same [Z] matrix as expressed in (31), (40) and (43). We then solve (C-1) to obtain $\vec{I}$, the distribution of the current on the wire.

However, instead of using the way described above, we prefer to consider the dipole-wire problem as a special case of slot-wire problem. We are then able to check if our theory is correct.

This can be done by letting the magnetic current on the slot be the same as the dipole. If the slot is small enough, the magnetic current that represents it looks like a dipole. Then, solving $\vec{I}$ from (20), we obtain the solution of the dipole-wire problem.

Mathematically, this procedure is as follows:

1. Let the length of the slot be very small, say $b/\lambda = 0.001$.

2. Let

$$-\int_{\text{slot}} M\,dy = K\ell = 1 \tag{C-3}$$

where -M is the axial magnetic current on the slot in region b. Recalling (8), we may set

$$NA = 1 \tag{C-4}$$

$$V_1 = 2/b \tag{C-5}$$

for the simplest case. Equation (C-5) results from the fact that the integral of the triangle equals b/2.

3. Solve (20) for $\vec{I}$, where $[\hat{T}]$, $[Z]$ are defined as before.

Again, the coefficient α_1 represents the magnitude of outgoing traveling current, and the other α's represent the higher order mode currents on the wire. We compute the current I from (20) for several cases. The results are given in Table C-1, along with those obtained by using Kajfez's formulation [1].

Table C-1. Magnitude of the outward traveling current induced on a wire by a magnetic dipole. Units of $10^{-2}/\lambda K\ell$.

d/λ	Our Results	Kajfez's Results
0.05	-1.1522 +j0.0423	-1.1520
0.078125	-0.6727 +j0.02737	-0.6721
0.10	-0.5011 +j0.02894	-0.5007
0.15	-0.3085 +j0.03392	-0.3100
0.20	-0.2209 +j0.02217	-0.2214
0.25	-0.1664 +j0.02352	-0.1707

Table C-1 shows that our results agree closely with Kajfez's results except for the small imaginary part. This small imaginary part must be due to numerical error in our solution.

PART TWO

COMPUTER PROGRAMS

I Introduction

The program used to compute the examples of this report is described and listed in this part. This program consists of subroutines ZT, YH, MUL, SICI, GAUSS, TOE, Subprogram function P and a main program. Each is accompanied by an explanation.

II General Impedance Matrix

Subroutine ZT(NB, WL, A, DD, Z) computes the generalized Z-matrix for an infinite thin wire behind an infinite conducting plane. The input variables are defined as

NB = the number of electric current expansion functions on the wire.

WL = the length (in wavelengths) where the higher order mode currents are considered to exist.

A = the radius (in wavelengths) of the wire.

DD = the distance (in wavelengths) from the wire to the conducting plane.

The output is stored in two dimension array Z.

Minimum allocations are given by

Complex Z (NB, NB), PP (NP)

where

$$NP = 2(NB + 1)$$

As described in Part One, the electric current expansion functions are divided into two parts. One is the outward traveling currents which travel from the origin to infinity. The others are higher order mode currents which are assumed to exist in the region $|X| < WL/2$. Therefore the resulting Z matrix is formed by adding one column and one row to the Z matrix for an open wire of length WL behind a conducting plane. DO loop 10 computes this Z matrix and stores the elements in Z(I+1, J+1). The additional row and column are computed by DO loop 20, and assigned as the first row and the first column of the resulting Z matrix.

Listing of ZT

```
      SUBROUTINE ZT(NB,WL,A,DD,Z)
      COMPLEX Z(50,50),CJ,P,PP(102),Z1,Z2,P2
      DIMENSION C(4),D(4)
      COMMON PI,AK,CJ,C
      L=2*NB
      N=NB-1
      DX=WL/L
      D(1)=0.5/DX
      D(2)=D(1)
      D(3)=-D(1)
      D(4)=D(3)
      AA=2.*DD
      K=L-1
      K1=2*(NB+1)
      DO 3 I=1,K1
      X=I-1.
3     PP(I)=P(K,WL,A,X,0.)-P(K,WL,AA,X,0.)
      DO 10 I=1,N
      DO 10 J=1,I
      Z1=(0.,0.)
      Z2=(0.,0.)
      DO 5 KA=1,4
      DO 5 KB=1,4
      IA=2*I-2+KA
      JB=2*J-2+KB
      M=1+IABS(IA-JB)
      Z1=Z1+C(KA)*C(KB)*PP(M)
5     Z2=Z2+D(KA)*D(KB)*PP(M)
      Z(I+1,J+1)=CJ*DX*DX*(240*PI*PI*Z1-60.*Z2)
      Z(J+1,I+1)=Z(I+1,J+1)
10    CONTINUE
      E=240.*PI*DX
```

```
      XN=N+1.5
      DO 20 M=1,N
      Z1=(0.,0.)
      DO 15 KA=1,4
      AM=2*M-2+KA
      P2=P(K,WL,A,AM,XN)-P(K,WL,AA,AM,XN)
      Z1=Z1+C(KA)*P2
15    CONTINUE
      Z(M+1,1)=E*Z1
20    Z(1,M+1)=Z(M+1,1)
      T=AK*A
      CALL SICI(SI,CI,T)
      Z2=-CI+CJ*SI
      Y=AK*AA
      CALL SICI(SI1,CI1,Y)
      Z2=Z2+CI1-CJ*SI1
      Z(1,1)=120.*Z2
      RETURN
      END
```

III Generalized Admittance Matrix

The half space generalized admittance matrix of the slot is a symmetric Toeplitz matrix, due to the symmetry of the slot and equal segments. This means that the Y matrix is of the form as

$$[Y] = \begin{bmatrix} Y_1 & Y_2 & \cdots & Y_n \\ Y_2 & Y_1 & \cdots & Y_{n-1} \\ \vdots & & & \\ Y_n & Y_{n-1} & \cdots & Y_1 \end{bmatrix}$$

Hence, we need only evaluate one column or one row of the [Y] matrix.

Subroutine YH(NA, B, W, Y) computes the first column of the half-space admittance matrix for a narrow slot. The input variables are defined as

NA = the number of axial magnetic current expansion functions on the slot.

B = the length (in wavelengths) of the slot.

W = the width (in wavelengths) of the slot.

The output is stored in one dimension array Y.

Minimum allocations are given by

Complex Y(NA), PP(NY)

where

$$NY = 2(NA + 1)$$

DO loop 3 is used to compute the ψ functions and stores them in PP so that repeated computation can be avoided.

Listing of YH.

```
      SUBROUTINE YH(NA,B,W,Y)
      COMPLEX Y(50),CJ,P,PP(102),Y1,Y2
      DIMENSION C(4),D(4)
      COMMON PI,AK,CJ,C
      A=W/4.
      K=2*NA+1
      DY=B/(K+1)
      D(1)=0.5/DY
      D(2)=D(1)
      D(3)=-D(1)
      D(4)=D(3)
      K1=2*(NA+1)
      DO 3 I=1,K1
      X=1.*(I-1)
3     PP(I)=P(K,B,A,X,0.)
      DO 10 I=1,NA
      Y1=(0.,0.)
      Y2=(0.,0.)
      DO 5 KA=1,4
      DO 5 KB=1,4
      IA=2*I-2+KA
      M=1+IABS(IA-KB)
      Y1=Y1+C(KA)*C(KB)*PP(M)
5     Y2=Y2+D(KA)*D(KB)*PP(M)
10    Y(I)=CJ*DY*DY*(Y1/30.-Y2/(120.*PI*PI))
      RETURN
      END
```

IV. Subroutine TOE

For a symmetric Toeplitz matrix we have a recursive system of equations [10] as follows:

$$\psi_1^{(1)} = Y_1^{-1} Y_2$$

$$\Delta^{(1)} = Y_1^{-1}$$

$$\Delta^{(m+1)} = [1 - (\psi_m^{(m)})^2]^{-1} \Delta^{(m)}$$

$$\psi_{m+1}^{(m+1)} = -\Delta^{(m+1)} \left(\sum_{S=1}^{m} \psi_S^{(m)} Y_{m-S+2} - Y_{m+2} \right)$$

$$\psi_{r+1}^{(m+1)} = \psi_{r+1}^{(m)} - \psi_{m-r+1}^{(m)} \psi_{m+1}^{(m+1)} \qquad 1 \le r \le m$$

for m = 1,2,...,N. Then the N×N matrix $[Z] = [Y]^{-1}$ is defined by

$$\begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1N} \\ Z_{21} & Z_{22} & \cdots & Z_{2N} \\ \vdots & & & \\ Z_{N1} & Z_{N2} & \cdots & Z_{NN} \end{bmatrix}$$

where the elements of [Z] are given by

$$Z_{11} = \Delta^{(N)}$$

$$Z_{r1} = -\psi_{r-1}^{(N-1)} \cdot \Delta^{(N)} \qquad 2 \le r \le N-1$$

$$Z_{rs} = Z_{r-1,s-1} + \psi_{r-1}^{(N-1)} \Delta^{(N)} \psi_{s-1}^{(N-1)} - \psi_{N-r+1}^{(N-1)} \Delta^{(N)} \psi_{N-S+1}^{(N-1)}$$

$$2 \le r, \quad S \le N$$

Note that the inverse matrix [Z] of [Y] is also symmetrical, so we need only compute the elements of the lower left triangle, and then set the upper right triangle equal to the transpose of the lower left triangle.

Subroutine TOE (N, Y, Z) is used to invert a symmetrical Toeplitz matrix [Y]. The input variables are defined as

N = the order of the matrix to be inverted

Y(N) = the first column of the matrix to be inverted.

The output is the inverse matrix of [Y] and is stored in two dimension array Z(N,N).

Minimum allocations are given by

Complex Z(N,N), Y(N), PS(N,N), DL(N)

DO loop 40 evaluates the half elements of [Z], except for Z_{11}, which is evaluated by statement Z(1,1) = DL(N). The other half elements are given by the statement Z(J,I) = Z(I,J).

Listing of TOE.

```
      SUBROUTINE TOE(N,Y,Z)
C     THIS IS YHE PROGRAM TO INVERT A TOEPLITZ MATRIX Y(NXN) INTO Z(NXN)
      COMPLEX Z(50,50),Y(50),A,PS(50,50),DL(50)
      N1=N-1
      N2=N1-1
      DL(1)=1./Y(1)
      PS(1,1)=DL(1)*Y(2)
      DO 20 M=1,N2
      M1=M+1
      DL(M1)=DL(M)/(1.-PS(M,M)*PS(M,M))
      A=(0.,0.)
      DO 10 K=1,M
10    A=A+PS(M,K)*Y(M-K+2)
      PS(M1,M1)=-DL(M1)*(A-Y(M+2))
      DO 30 K=1,M
30    PS(M1,K)=PS(M,K)-PS(M,M1-K)*PS(M1,M1)
20    CONTINUE
      DL(N)=DL(N1)/(1.-PS(N1,N1)*PS(N1,N1))
      Z(1,1)=DL(N)
      DO 40 I=2,N
      I1=I-1
```

```
      Z(I,1)=-PS(N1,I1)*DL(N)
      Z(1,I)=Z(I,1)
      DO 40  J=2,I
      Z(I,J)=Z(I1,J-1)+PS(N1,I1)*DL(N)*PS(N1,J-1)-PS(N1,N-I+1)*DL(N)
     1 *PS(N1,N-J+1)
40    Z(J,I)=Z(I,J)
      RETURN
      END
```

V. Subroutine GAUSS

Subroutine GAUSS (N, A, B, EPS, ISW) is used to solve a linear system of equations with complex coefficients by the method of Gaussian elimination. The coefficients of unknowns are stored in two dimension array A, and the constants are stored in one dimension array B. Again the solution of the equations are stored in B. N is the number of unknowns as well as the equations. EPS is a small constant and ISW is a message. If the absolute value of the pivot of column is larger than EPS, ISW equals to 1. Otherwise ISW = 0.

Minimum allocations are given by

Complex A(N,N) B(N)

Listing of GAUSS

```
      SUBROUTINE GAUSS(N,A,B,EPS,ISW)
C     TO SOLVE EQUATIONS AX=B
      COMPLEX A(50,50),B(50),C,T
      NM1=N-1
      DO 10  K=1,NM1
      C=(0.,0.)
      DO 2  I=K,N
      IF(CABS(A(I,K)).LE.CABS(C)) GO TO 2
      C=A(I,K)
      IO=I
2     CONTINUE
      IF(CABS(C).GE.EPS) GO TO 3
      ISW=0
      RETURN
3     IF(IO.EQ.K) GO TO 6
      DO  4  J=K,N
      T=A(K,J)
      A(K,J)=A(IO,J)
4     A(IO,J)=T
```

```
      T=B(K)
      B(K)=B(IO)
      B(IO)=T
6     KP1=K+1
      C=1./C
      B(K)=B(K)*C
      DO 10 J=KP1,N
      A(K,J)=A(K,J)*C
      DO 20 I=KP1,N
20    A(I,J)=A(I,J)-A(I,K)*A(K,J)
10    B(J)=B(J)-A(J,K)*B(K)
      B(N)=B(N)/A(N,N)
      DO 40 K=1,NM1
      I=N-K
      C=(0.,0.)
      IP1=I+1
      DO 50 J=IP1,N
50    C=C+A(I,J)*B(J)
40    B(I)=B(I)-C
      ISW=1
      RETURN
      END
```

VI Subroutine MUL

Subroutine MUL(L, M, N, A, B, C) is used to multiply matrix A by matrix B and stores the result in C. Here A is an $(L \times M)$ array, B is an $(M \times N)$ array and C is an $(L \times N)$ array.

Minimum allocations are given by

Complex A(L, M), B(M, N), C(L, N)

Listing of MUL.

```
      SUBROUTINE MUL(L,M,N,A,B,C)
C     TO DO MULTIPLICATION C=A*B
      COMPLEX A(50,50),B(50,50),C(50,50),W
      DO 20 I=1,L
      DO 20 K=1,N
      W=(0.,0.)
      DO 10 J=1,M
10    W=A(I,J)*B(J,K)+W
20    C(I,K)=W
      RETURN
      END
```

VII Subrotuine CISI

Subroutine CISI (SI, CI, X) is used to compute the sine and cosine integrals.

$$Ci(x) = -\int_{-X}^{\infty} \frac{\cos v}{v}\, dv$$

$$Si(x) = \int_{0}^{X} \frac{\sin v}{v}\, dv$$

This subroutine is described in [11].

Listing of CISI.

```
      SUBROUTINE SICI(SI.CI.X)
      Z=ABS(X)
      IF(Z-4.) 1.1.4
1     Y=(4.-Z)*(4.+Z)
      SI=X*((((((1.753141E-9*Y+1.568988E-7)*Y+1.374168E-5)*Y+6.939889
     1 E-4)*Y+1.964882E-2)*Y+4.395509E-1)
      CI=((5.772156E-1+ALCG(Z))/Z-Z*(((((1.386985E-10*Y+1.584996E-8)*Y
     2 +1.725752E-6)*Y+1.185999E-4)*Y+4.99092E-3)*Y+1.315308E-1))*Z
      RETURN
4     SI=SIN(Z)
      Y=COS(Z)
      Z=4./Z
      U=(((((((((4.048069E-3*Z-2.279143E-2)*Z+5.51507E-2)*Z-7.261646E-2)
     0 *Z+4.987716E-2)*Z-3.332519E-3)*Z-2.314617E-2)*Z-1.134958E-5)*Z
     4 +6.250011E-2)*Z+2.583989E-10
      V=((((((((((-5.108699E-3*Z+2.819179E-2)*Z-6.537283E-2)*Z
     5 +7.902034E-2)*Z-4.400416E-2)*Z-7.945556E-3)*Z+2.601293E-2)*Z
     L -3.764E-4)*Z-3.122418E-2)*Z-6.646441E-7)*Z+2.5E-1
      CI=Z*(SI*V-Y*U)
      SI=-Z*(SI*U+Y*V)+1.570796
      RETURN
      END
```

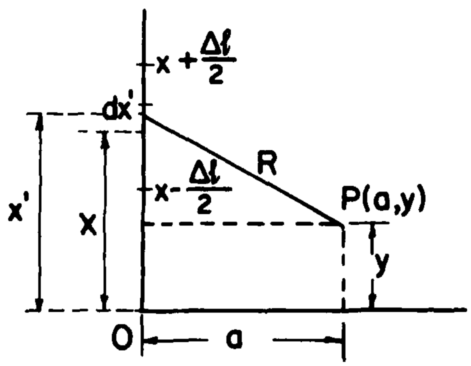
VIII Subprogram Function P

An accurate evaluation of the scalar Green's function of (B-21) is developed in [9].

Subprogram complex function P(N, B, A, X, Y) computes the function

$$\psi = \frac{1}{4\pi\Delta\ell} \int_{X-\Delta\ell/2}^{X+\Delta\ell/2} \frac{e^{-jkR}}{R}\, dx'$$

where $\Delta\ell = b/(n+1)$ is the integral length and $R = \sqrt{a^2 + (x'-y)^2}$ is the distance from source point dx' to the field point. This is shown in the following figure.



With reference to the figure, the input variables are defined as.

N = n, an integer

B = b, a length (in wavelengths) which gives the subsection interval $\Delta\ell = b/(N+1)$.

A = a, the transverse coordinate of the field point.

X = x, the longitudinal coordinate of the source point.

Y = y, the longitudinal coordinate of the field point.

Note that the integrand is expanded in a Taylor series for R < 5$\Delta\ell$, and in a Maclaurin series for R $\geq$ 5$\Delta\ell$. This is done by the statement

IF (RR. GE. 10) GO TO 100.

Listing of subprogram function P.

```
      COMPLEX FUNCTION P(N,B,A,X,Y)
C     THIS IS THE PROGRAM TO CALCULATE THE INTEGRAL EXP(-JKR)/R
      COMPLEX T,J,TT ,CEXP
      PI=3.14159265
      AK=2.*PI
      J=(0.,1.)
      D=B/(N+1)
      AL=D/2.
      DZ=D*ABS(X-Y)
      R=SQRT(A*A+DZ*DZ)
      RR=R/AL
      IF(RR.GE.10) GO TO 100
      G=SQRT(A*A+(DZ-AL)*(DZ-AL))
      H=SQRT(A*A+(DZ+AL)*(DZ+AL))
      IF(DZ.GE.AL) GO TO 9
      B1=ALOG(((DZ+AL+H)*(G+AL-DZ))/(A*A))
      GO TO 99
9     B1=ALOG((DZ+AL+H)/(DZ-AL+G))
99    B3=(DZ+AL)/2.*H+(AL-DZ)/2.*G+A*A*B1/2.
      B4=2.*AL*A*A+(2*AL**3+6.*AL*DZ*DZ)/3.
      T=CEXP(-J*AK*R)/(8.*PI*AL)
      V=AK**3/6.*(B4-3.*R*B3+3.*R*R*D-R**3*B1)
      P=T*(B1-J*AK*(D-R*B1)-AK*AK/2.*(B3-2.*R*D+R*R*B1)+J*V)
      GO TO 200
100   C=AL/R
      C2=C*C
      E=DZ/R
      E2=E*E
      E4=E2*E2
      A1=C/6.*(3.*E2-1)+C*C2*(3.-30.*E2+35.*E4)/40.
      A0=1.+C*A1
      A2=-E2/6.-C2/40.*(1-12.*E2+15.*E4)
      A3=C/60.*(3.*E2-5.*E4)
      A4=E4/120.
      BK=AK*AL
      BK2=BK*BK
      TT=CEXP(-J*AK*R)/(4.*PI*R)
      P=TT*(A0+J*BK*A1+BK2*A2+J*BK2*BK*A3+BK2*BK2*A4)
200   RETURN
      END
```

IX Main Program

The main program computes the complex coefficients, V_n which determine the axial magnetic current on the slot, α_n which determine the electric currents on the wire, and the equivalent circuit parameters of the transmission line. The main program calls the subroutines ZT, YH, TOE, MUL, GAUSS and CISI, which are described previously in Part 2.

One data card is read in the main program according to

```
      READ (1,8) NA, NB, B, WL, W
    8 FORMAT (2I2, 3F4.3)
```

The input data are defined as

NA = the number of magnetic current expansion functions.

NB = the number of electric current expansion functions.

B = the length of the slot (in wavelengths).

WL = the length (in wavelengths) near the slot on the wire where the higher order mode currents are considered to exist.

W = the width of the slot (in wavelengths).

Another input, the distance D, is assigned by DO loop 200.

Minimum allocations are given by

```
Complex T(NA, NB), TT(NB, NA), Z(NB, NB),
        YZ(NA,NA), GZ(NB, NB), YT(NA, NB),
        VM(NA), VI(NB), Y(NA), YTI(NA)
```

DO loop 40 computes the [T] and $[\hat{T}]$ matrices. DO loop 60

is used to reduce the order of matrix equations, NB by NB to $(\frac{NB+3}{2})$ by $(\frac{NB+3}{2})$, if NB is odd, or $(\frac{NB}{2} + 1)$ by $(\frac{NB}{2} + 1)$, if NB is even. This is due to the symmetry of the problem. It is done to save computation. DO loop 120 computes the distribution of currents twice. The first time is for incidence from region b, the incident wave being a TEM traveling wave. The second time is for incidence from region a, the incident wave being a plane wave. The current solutions are printed out each time because they are not kept in permanent storage.

In this main program, we compute only the case $y_c/\lambda = 0$. This is done by assign statement early

$$YC = 0$$

Listing of main program.

```
$JOB            NAIHENG
C      THE MAIN PROGRAM TO SOLVE SLOT-WIRE PROBLEM
       COMPLEX T(50,50),TT(50,50),Z(50,50),YZ(50,50),GZ(50,50)
      1 ,YI(50),VS(50),VM(50),VI(50),Y(50),CJ,E1,E2,H,CEXP,YE,
      2 DGZ(50,50),YTI(50),GG(50,50),YT(50,50),V0,AI(2)
       DIMENSION C(4)
       EQUIVALENCE (DGZ,T),(GG,GZ)
       COMMON PI,AK,CJ,C
       PI=3.14159265
       AK=2*PI
       CJ=(0.,1.)
       A=0.001
       YC=0.
       READ(1,8) NA,NB,B,WL,W
8      FORMAT(2I2,3F4.3)
       DO 200 NN=1,2
       D=0.1*NN
       WRITE(3,9) NA,B,W
9      FORMAT(//1X,'NA=',I2,3X,'B=',F6.4,3X,'W=',F6.4)
       WRITE(3,10) NB,WL,A,D
10     FORMAT(1X,'NB=',I2,3X,'WL=',F6.4,3X,'A=',F6.4,3X,'D=',F6.4)
C      A=THE RADIUS OF THE WIRE
C      W=THE WIDTH OF THE SLOT
C      NA=THE NUMBER OF EXPANSION FUNCTIONS ON THE SLOT
C      NB=THE NUMBER OF EXPANSION FUNCTIONS ON THE WIRE
C      B=THE LENGTH OF THE SLOT
C      D=THE DISTANCE BETWEEN THE SCREEN AND THE WIRE
```

```
      C(1)=0.25
      C(2)=0.75
      C(3)=C(2)
      C(4)=C(1)
      DX=WL/(2*NB)
      DY=B/(2*NA+2)
      CALL ZT(NB,WL,A,D,Z)
      CALL YH(NA,B,W,Y)
C     Y=ADMITTANCE MATRIX OF A SLOT IN HALF SPACE
      DO 12 I=1,NA
12    Y(I)=2.*Y(I)
      CALL TOE(NA,Y,YZ)
C     TO COMPUTE COUPLE MATRICES,T AND TT
15    WRITE(3,20) YC
20    FORMAT(///3X,'YC=',F4.2)
      DO 40 M=1,NA
      N1=NB-1
      DO 30 N=1,N1
      E1=(0.,0.)
      E2=(0.,0.)
C     VS=THE EXCITATION VECTOR
      VS(M)=(0.,0.)
      DO 25 KA=1,4
      MA=2*M-2+KA
      YY=(2*MA-1)*DY/2.-B/2.-YC
      YP=(YY+DY/2.)/D
      YN=(YY-DY/2.)/D
      AT=ATAN(YP)-ATAN(YN)
      VS(M)=VS(M)-C(KA)*AT/PI
      RY=SQRT(YY*YY+D*D)
      H=CEXP(-CJ*AK*RY)
      E2=E2+C(KA)*H/(RY*RY)
      DO 25 KB=1,4
      NKB=2*N-2+KB
      X=(2*NKB-1)*DX/2.-WL/2.
      R=SQRT(X*X+YY*YY+D*D)
      E1=E1+C(KA)*C(KB)*(1/R**3+CJ*AK/(R*R))*CEXP(-CJ*AK*R)
25    CONTINUE
      T(M,N+1)=D*DX*DY*E1/AK
      TT(N+1,M)=-T(M,N+1)
30    CONTINUE
      T(M,1)=D*DY*E2/PI
      TT(1,M)=-T(M,1)
40    CONTINUE
      CALL MUL(NA,NA,NB,YZ,T,YT)
      CALL MUL(NB,NA,NB,TT,YT,GZ)
      IN=NB/2
      L=IN+1
      DO 60 I=1,L
      DGZ(I,1)=GZ(I,1)-Z(I,1)
      DO 60 J=2,L
      J1=NB-J+2
```

```
      DGZ(I,J)=GZ(I,J)-Z(I,J)+GZ(I,J1)-Z(I,J1)
      IF(2*IN.EQ.NB) GO TO 55
      GO TO 60
55    DGZ(I,L)=GZ(I,L)-Z(I,L)
60    CONTINUE
      DO 120 K=1,2
      DO 65 I=1,L
      DO 65 J=1,L
      GG(I,J)=DGZ(I,J)
65    CONTINUE
      IF(K.EQ.1) GO TO 85
      WRITE(3,130)
      DO 75 I=1,NA
75    VS(I)=4.*DY
      GO TO 90
85    WRITE(3,125)
90    CONTINUE
      DO 95 I=1,NA
      YI(I)=(0.,0.)
      DO 95 J=1,NA
95    YI(I)=YZ(I,J)*VS(J)+YI(I)
      DO 100 I=1,NB
      VI(I)=(0.,0.)
      DO 100 J=1,NA
100   VI(I)=TT(I,J)*YI(J)+VI(I)
      CALL GAUSS(L,GG,VI,1E-11,ISW)
      IF(ISW.EQ.1) GO TO 102
      WRITE(3,101)
101   FORMAT('  ISW=0')
      STOP
102   WRITE(3,135)
      DO 103 I=2,L
103   VI(NB-I+2)=VI(I)
      WRITE(3,140) VI(1)
      WRITE(3,145)
      WRITE(3,160)(VI(I),I=2,NB)
      DO 105 I=1,NA
      YTI(I)=(0.,0.)
      DO 105 J=1,NB
105   YTI(I)=YT(I,J)*VI(J)+YTI(I)
      DO 110 I=1,NA
      VM(I)=YI(I)-YTI(I)
      VM(I)=VM(I)/376.99
110   CONTINUE
      WRITE(3,150)
      WRITE(3,160)(VM(I),I=1,NA)
120   AI(K)=VI(1)
      Z0=60.*ALOG(2.*D/A)
      YE=-(1+1/AI(1))/(2.*Z0)
      V0=-AI(2)*(Z0+1/(2.*YE))
      V0=2*V0
```

```
      WRITE(3,165)
      WRITE(3,170) YE,VO
      REY=REAL(1./YE)
      RV=CABS(VO)
      PL=RV*RV/(2*REY)
      WRITE(3,180) PL
200   CONTINUE
125   FORMAT(///' INCIDENT WAVE IS TEM WAVE')
130   FORMAT(///' INCIDENT WAVE IS PLANE WAVE')
135   FORMAT(/' ELECTRIC CURRENT ON THE WIRE')
140   FORMAT(/3X,'THE OUTGOING TRAVELING CURRENT='/3X,2F16.6)
145   FORMAT(/3X,'NON-TEM CURRENTS ARE')
150   FORMAT(/' AXIAL MAGNETIC CURRENT ON THE SLOT=')
160   FORMAT(3X,2F16.6)
165   FORMAT(//' THE EQUIVALENT NETWORK PARAMETERS')
170   FORMAT(4X,'Y=',2E16.6/3X,'VO=',2E16.6)
180   FORMAT(' THE POWER TRANSMITTED P=',E15.6)
      STOP
      END
```

```
          $DATA

 NA=21   B=0.5000   W=0.0500
 NB=32   WL=1.6000   A=0.0010   D=0.1000


   YC=0.00


 INCIDENT WAVE IS TEM WAVE

 ELECTRIC CURRENT ON THE WIRE

   THE OUTGOING TRAVELING CURRENT=
          -0.205364         0.080070

   NON-TEM CURRENTS ARE
          -0.000876        -0.004263
          -0.001889        -0.007896
          -0.002893        -0.011146
          -0.003648        -0.013871
          -0.003886        -0.015925
          -0.003312        -0.017164
          -0.001687        -0.017454
           0.001205        -0.016653
           0.005511        -0.014597
           0.011303        -0.011058
           0.018590        -0.005676
           0.027366         0.002187
           0.037724         0.013782
```

0.050119	0.031745
0.066235	0.062589
0.082429	0.100017
0.066235	0.062589
0.050119	0.031745
0.037724	0.013782
0.027366	0.002187
0.018590	-0.005676
0.011303	-0.011058
0.005511	-0.014597
0.001205	-0.016653
-0.001687	-0.017454
-0.003312	-0.017164
-0.003880	-0.015925
-0.003648	-0.013871
-0.002893	-0.011146
-0.001889	-0.007896
-0.000876	-0.004263

AXIAL MAGNETIC CURRENT ON THE SLOT=

-0.166122	0.071415
-0.224527	0.092303
-0.299857	0.118933
-0.360378	0.137290
-0.415273	0.151804
-0.461746	0.161629
-0.500140	0.167483
-0.529987	0.169945
-0.551210	0.170065
-0.563838	0.169204
-0.568021	0.168718
-0.563838	0.169204
-0.551210	0.170065
-0.529987	0.169945
-0.500140	0.167483
-0.461746	0.161629
-0.415272	0.151804
-0.360378	0.137289
-0.299856	0.118933
-0.224527	0.092303
-0.166121	0.071415

INCIDENT WAVE IS PLANE WAVE

ELECTRIC CURRENT ON THE WIRE

THE OUTGOING TRAVELING CURRENT=

0.225198	-0.091860

NON-TEM CURRENTS ARE

0.001035	0.004714
0.002198	0.008733
0.003329	0.012325
0.004162	0.015331
0.004397	0.017587
0.003732	0.018932
0.001890	0.019216
-0.001353	0.018285
-0.006148	0.015959
-0.012563	0.011992
-0.020602	0.005995
-0.030252	-0.002728
-0.041627	-0.015538
-0.055279	-0.035318
-0.073243	-0.069199
-0.091586	-0.110264
-0.073243	-0.069199
-0.055279	-0.035318
-0.041627	-0.015538
-0.030252	-0.002728
-0.020602	0.005995
-0.012563	0.011992
-0.006148	0.015959
-0.001353	0.018285
0.001890	0.019216
0.003732	0.018932
0.004397	0.017587
0.004162	0.015331
0.003329	0.012325
0.002198	0.008733
0.001035	0.004714

AXIAL MAGNETIC CURRENT ON THE SLOT=

0.182180	-0.054367
0.246201	-0.076092
0.328772	-0.103953
0.395092	-0.127723
0.455233	-0.150033
0.506132	-0.169758
0.548168	-0.186742
0.580834	-0.200546
0.604052	-0.210811
0.617862	-0.217170
0.622435	-0.219330
0.617862	-0.217170
0.604052	-0.210812
0.580834	-0.200546
0.548168	-0.186742
0.506131	-0.169758
0.455232	-0.150033
0.395091	-0.127723
0.328771	-0.103953
0.246200	-0.076092
0.182179	-0.054367

```
THE EQUIVALENT NETWORK PARAMETERS
   Y=     0.507528E-02     0.259205E-02
  V0=    -0.171041E 03     0.907327E 02
THE POWER TRANSMITTED P=    0.119943E 03


NA=21    B=0.5000    W=0.0500
NB=32    WL=1.6000    A=0.0010    D=0.2000


  YC=0.00

INCIDENT WAVE IS TEM WAVE

ELECTRIC CURRENT ON THE WIRE

  THE OUTGOING TRAVELING CURRENT=
         -0.081129         0.060615

  NON-TEM CURRENTS ARE
         -0.001276         -0.001675
         -0.002684         -0.003272
         -0.004092         -0.004987
         -0.005232         -0.006844
         -0.005788         -0.008815
         -0.005431         -0.010814
         -0.003840         -0.012685
         -0.000731         -0.014189
          0.004119         -0.014994
          0.010866         -0.014665
          0.019595         -0.012641
          0.030348         -0.008211
          0.043154         -0.000484
          0.058049          0.011584
          0.075240          0.029302
          0.089693          0.046771
          0.075240          0.029302
          0.058049          0.011584
          0.043154         -0.000484
          0.030348         -0.008211
          0.019595         -0.012641
          0.010866         -0.014665
          0.004119         -0.014994
         -0.000731         -0.014189
         -0.003840         -0.012685
         -0.005431         -0.010814
         -0.005788         -0.008815
         -0.005232         -0.006844
         -0.004092         -0.004987
         -0.002684         -0.003272
         -0.001276         -0.001675
```

AXIAL MAGNETIC CURRENT ON THE SLOT=

-0.113120	0.069438
-0.153494	0.092154
-0.205619	0.121465
-0.247950	0.144032
-0.286674	0.163980
-0.319843	0.180350
-0.347610	0.193531
-0.369537	0.203555
-0.385399	0.210579
-0.394995	0.214727
-0.398207	0.216098
-0.394995	0.214727
-0.385399	0.210579
-0.369536	0.203555
-0.347609	0.193531
-0.319842	0.180350
-0.286673	0.163980
-0.247949	0.144032
-0.205619	0.121465
-0.153493	0.092154
-0.113120	0.069438

INCIDENT WAVE IS PLANE WAVE

ELECTRIC CURRENT ON THE WIRE

THE OUTGOING TRAVELING CURRENT=

0.128560	-0.096623

NON-TEM CURRENTS ARE

0.002036	0.002664
0.004275	0.005203
0.006510	0.007930
0.008312	0.010879
0.009185	0.014005
0.008605	0.017170
0.006064	0.020127
0.001117	0.022496
-0.006590	0.023754
-0.017301	0.023208
-0.031153	0.019973
-0.048214	0.012924
-0.068537	0.000652
-0.092191	-0.018494
-0.119527	-0.046586
-0.142539	-0.074273
-0.119527	-0.046586
-0.092191	-0.018494
-0.068537	0.000652
-0.048214	0.012924

-0.031153	0.019973
-0.017301	0.023208
-0.006590	0.023754
0.001117	0.022496
0.006064	0.020127
0.008605	0.017170
0.009185	0.014005
0.008312	0.010879
0.006510	0.007930
0.004275	0.005203
0.002036	0.002664

AXIAL MAGNETIC CURRENT ON THE SLOT=

0.179188	-0.096876
0.243137	-0.133158
0.325701	-0.179775
0.392747	-0.218353
0.454081	-0.253917
0.506615	-0.284658
0.550593	-0.310557
0.585321	-0.331120
0.610444	-0.346054
0.625641	-0.355113
0.630727	-0.358149
0.625640	-0.355113
0.610443	-0.346054
0.585320	-0.331120
0.550592	-0.310557
0.506615	-0.284657
0.454080	-0.253917
0.392747	-0.218353
0.325700	-0.179775
0.243136	-0.133157
0.179188	-0.096876

THE EQUIVALENT NETWORK PARAMETERS

Y= 0.961138E-02 0.822018E-02

V0= -0.951908E 02 0.818828E 02

THE POWER TRANSMITTED P= 0.131187E 03

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